We say a triangle in the coordinate plane is integral if its three vertices have integer coordinates and if its three sides have integer length. There are two questions:
(a) Find an integral triangle with perimeter of 42, and
(b) Is there an integral triangle with perimeter of 43? If not, why not?

(a) Let's say we have an integral triangle. Notice that if we move it "down one," i.e. we decrease all it's coordinates by 1, then we still have an integral triangle. We can also move it "up one," "left one," or "right one" and still have an integral triangle. (We can also do some mirror images, but we won't need those.) This means that, if there's one integral triangle, there's another one that has one of its corners at the origin. Let's try and find that one.

It's a bit of work. Let's break our search up into cases. Our triangle can have either 1,2, or 3 diagonal sides (sides not parallel to the axes). If it has just 1 diagonal side, then it is a right triangle, and we may assume that the right angle corner is at the origin. Unfortunately, all of the right triangle "Pythagorean triples" with perimeters less than 43 are given below, and none of them have perimeter 42.

$$(3,4,5),(5,12,13),(6,8,10),(8,15,17),(9,12,15)$$

So we move on. If the triangle has 2 diagonal sides, then those two sides must be hypotenuses of a right triangle (and so must be the last number of a Pythagorean triple), and the other side must be parallel to an axis. Let's say that that non-diagonal side is parallel to the x-axis, and that the left corner of that side is at the origin. (So the non-diagonal side is in fact on the x-axis.) Notice that in this case the two right triangles share a (non-diagonal) side. Now there are a few more Pythagorean triples that have hypotenuses less than 42 that we can add to the list above, but let's check the only two that have equal length sides first: the $(5,12,13)$ and $(9,12,15)$ ones. Eureka! When we put these two triangles "back-to-back" along the length 12 sides, we get a single triangle with side lengths of 15, 14 and 13, which total up to a perimeter of 42.

(b) There is no such integral triangle. Just as in part (a), any integral triangle with perimeter 43 can be moved so that one of its corners is the origin. Let's say our mythical integral triangle has coordinates $(0,0),(u,v)$, and (x,y). The lengths of our sides are

$$\sqrt{u^2+v^2}, \quad \sqrt{x^2+y^2}, \quad \sqrt{(u-x)^2+(v-y)^2}$$

Now since all of the triangle's sides are of integer length, that means that each side length is either even or odd. Since the perimeter 43 is odd, we must have two sides even or no sides even. Either way, since the square of an odd number is odd and the square of an even number is even, the odd–even pattern of the list above must be the same as the odd–even pattern of

$$u^2+v^2, \quad x^2+y^2, \quad (u-x)^2+(v-y)^2$$

and so these three numbers listed must add up to an odd number. But

$$u^2 + v^2 + x^2 + y^2 + (u - x)^2 + (v - y)^2 = 2u^2 + 2v^2 + 2x^2 + 2y^2 + 2uv + 2xy$$

is even! We've reached a contradiction, and there's no triangle that could possibly work.