General Examination: Graph Theory and Combinatorial Analysis

Problem 1.

- (a) Define $\pi : d_1 \leq d_2 \leq \ldots \leq d_n$ is realizable (as a graph).
- (b) Suppose π is realizable. Prove $\sum_{i=1}^{n} d_i = 2e$, where e is the number of edges.
- (c) Prove there exists $i \neq j$ such that $d_i = d_j$ for a graphical sequence.
- (d) Prove 444433 is realizable.

Problem 2.

- (a) Let G be a graph. Define: G is hamiltonian.
- (b) Suppose G is hamiltonian and S is a set of nodes. What can be said about |S| and the number of components of G S?
- (c) Consider the graph shown below where each of the n + 1 independent nodes is adjacent to each node in the K_n . Prove G is not hamiltonian.



(d) State Chvátal's degree condition for forcible hamiltonicity. What is special about the set of degree sequences satisfying the condition?

Problem 3.

- (a) State Menger's Theorem
- (b) State Hall's Marriage Theorem
- (c) Prove Hall's Marriage Theorem using Menger's Theorem

Problem 4.

- (a) Define: a difference table
- (b) Use a difference table to obtain closed functional expression for $\sum_{k=1}^n k^3$

Problem 5. Derive the number of spanning trees of K_n .