General Examination: Graph Theory and Combinatorial Analysis

Problem 1. Let G be a graph.

(a) Define $\kappa(G)$ – the (point) connectivity, $\lambda(G)$ – the line connectivity, and $\delta(G)$ – the minimum degree

(b) Give the relationships between them

(c) Prove:

$$\lambda(G) \leqslant \frac{2e(G)}{n(G)},$$

where $n(G)$ is the number of nodes in G and $e(G)$ is the number of edges

(d) Prove: if the diameter of G, $d(G)$, is $\leqslant 2$ then $\lambda = \delta$

Problem 2.

(a) Let G be a graph. Define: G is hamiltonian.

(b) If G is hamiltonian and S is a set of nodes, then what is the relationship between $|S|$ and the number of components of $G - S$? Prove your answer.

(c) State Ore's theorem for hamiltonicity

Problem 3.

(a) Define the norm of a difference table

(b) Use this norm to derive the closed formula for $\sum_{k=1}^{n} k^2$

Problem 4.

(a) State Generalized Inclusion / Exclusion

(b) Briefly describe one application of it