General Examination: Graph Theory and Combinatorial Analysis

Problem 1.

(a) State

- Hall's Marriage Theorem
- König-Egeváry Theorem (term rank/covering number)

(b) Use the König-Egeváry Theorem to prove Hall's Theorem

Problem 2.

(a) Determine the left edge of the difference table for $f(x) = x^3$

(b) Use (a) to derive the formula for $\sum_{k=1}^{n} k^3$

Problem 3.

(a) Define $R(p, q, 2)$

(b) Prove: $R(3, 3, 2) = 6$

Problem 4. Define κ – the (point) connectivity and λ – the line connectivity of a (simple) graph G

(a) Prove $\kappa \leqslant \lambda \leqslant \delta$ (the minimum degree)

(b) If $G = K_{p,q}$ find $\kappa(G)$ and $\lambda(G)$ – give reasons

Problem 5.

(a) Define: G is Eulerian

(b) State and prove the necessary and sufficient conditions for G to be Eulerian

Problem 6.

(a) Suppose π is a realizable degree sequence. Define: π is forcibly hamiltonian

(b) State Chvátal's condition for forcible hamiltonicity

(c) Chvátal's condition yields the largest upper order ideal of forcibly hamiltonian degree sequence. What is the meaning of this statement?