General Examination: Differential Equations

Problem 1. Consider the following predator-prey system

$$\frac{dx}{dt} = x(1-x) - axy,$$
$$\frac{dy}{dt} = -y + bxy,$$

where a and b are parameters with $0 < a < \infty$, $0 < b < \infty$.

1) Find all fixed points of the system.

2) Use linear analysis to classify the fixed points found in 1) when the method applies.

3) Describe the conditions for both predators and prey to coexist, and the conditions for only one of them to survive.

4) Plot the phase portrait when only one of the species survives (hint: you should use nullclines and the vector field for phase portrait).

5) Plot the phase portrait when both species coexist.

6) Use a phase portrait to classify those fixed points when linear analysis fails to do so.

7) Plot the regions with qualitatively different vector fields and the boundaries of these regions in the (a, b) plane, and then describe the types of the fixed points in each of these regions and classify the bifurcations on the boundaries.