Doctoral Program | **Department of Mathematical Sciences**
Spring 2011 | Stevens Institute of Technology

General Examination: Complex Analysis

Problem 1. (a) Find the constant c such that the function

$$f(z) = \frac{1}{z^9 - z^8 + z^2 + 2z - 3} - \frac{c}{z-1}$$

is holomorphic on an open set containing the closed disk $\{z : z \leq 1\}$ and prove it.

Problem 2.

(a) Let G be a region in $\mathbb{C}$ and suppose $u : G \to \mathbb{R}$ is a harmonic function. Show that $\partial u/\partial x - i\partial u/\partial y$ is an analytic function on G.

(b) Suppose that u and v are real valued harmonic functions on a domain Ω such that u and v satisfy the Cauchy-Riemann equations on a subset S of Ω which has a limit point in Ω. Using the result from part (a), show that $u + iv$ must be analytic on Ω.

Problem 3. Using techniques of complex analysis, evaluate the integrals

(a)

$$I = \int_{-\infty}^{\infty} \frac{\cos x}{\cosh x} dx.$$

(b)

$$I = \int_{0}^{\infty} \frac{x^{a-1}}{x+1} dx, \quad 0 < a < 1.$$