General Examination: Algebra and Linear Algebra

Problem 1. Let G and H be finite groups satisfying $\gcd(|G|,|H|) = 1$. Prove that $Aut(G \times H) \simeq Aut(G) \times Aut(H)$.

Problem 2. Let D be an integral domain and $D[x]$ the polynomial ring over D. Suppose $\phi : D[x] \to D[x]$ is a ring isomorphism such that $\phi(d) = d$ for all $d \in D$. Show that $\phi(x) = ax + b$ for some $a, b \in D$ such that a is a unit of D.

Problem 3. Show that a group of order $2 \cdot 7 \cdot 13$ must be solvable.