General Examination:Algebra

Do three of the following four problems.

1. Find the order of the element $(2,0)+\langle(4;4)\rangle$ in the factor group $(Z_6 \times Z_8)/\langle(4,4)\rangle$.
2. Let G be a group of order 30. Prove that G has a normal subgroup N with $1 \neq N \neq G$.
3. Denote the set of invertible elements of the ring Z/nZ by U_n.
   (a) List all the elements of U_{18}.
   (b) Is U_{18} a cyclic group under multiplication? Justify your answer.
4. Show that if R is a commutative ring with 1, but R is not a field, then the polynomial ring $R[x]$ is not a principal ideal domain.