Doctoral Program | **Department of Mathematical Sciences**
Spring 2009 | Stevens Institute of Technology

General Examination: Part I

Problem 1. Show that every finite group is isomorphic to a subgroup of an alternating group.

Problem 2. Let R be a commutative ring such that the polynomial ring $R[x]$ is a principal ideal domain. Prove that R is a field.

Problem 3. Let $(X,\mathfrak{I})$ be a topological space and let $\mathfrak{B}$ be the σ-algebra of Borel sets defined on the space.

(a) If $\mathscr{C}=\{U^c|\ U\in\mathfrak{I}\}$ is the family of all closed sets in X, show $\mathfrak{M}(\mathscr{C})=\mathfrak{B}$, where $\mathfrak{M}(\mathscr{C})$ is the σ-algebra generated by $\mathscr{C}$.

(b) Give an example of a Borel measurable function which is not continuous. Explain your answer.

Problem 4. Let $(X, \mathfrak{M})$ be a measure space with function $f : (X, \mathfrak{M}) \to [-\infty, \infty]$.

(a) Prove: f is measurable $\Longrightarrow |f|$ is measurable.

(b) Is the converse of (a) valid? If yes, prove it, if not give a counterexample.

Problem 5. Let C denote the positively oriented circle of radius 1 centered at the origin. Evaluate

$$\int_C e^{e^{1/z}}\,dz.$$

Problem 6. Determine all entire functions f that satisfy $|f'(z)| < |f(z)|$.