General Examination: Part II

Problem 1: Let R be a commutative ring with 1. For any $a \in R$ define the annihilator of a by $Ann(a) = \{r \in R \mid ar = 0\}$.

(a) Prove that $Ann(a)$ is an ideal.

(b) Suppose $Ann(a) \neq Ann(b)$ but both are maximal ideals. Show that $ab = 0$.

Problem 2:

(a) Show that the sum of all products of distinct integers taken two at a time from 1, 2, 3, …, n is

$$\frac{n(n-1)(n+1)(3n+2)}{24}.$$

(b) Find the sum of all products of distinct integers taken two at a time from 1, 3, 5, …, $2n-1$.

Problem 3: Determine all the complex analytic functions f defined on the unit disk D which satisfy

$$f''\left(\frac{1}{n}\right) + f\left(\frac{1}{n}\right) = 0$$

for $n = 2, 3, 4, \ldots$.

Problem 4: Show that

$$\sup_{\|f\|\leq 1,\, \|x\|\leq 1} |f(Ax)| = \|A\|.$$

(Hint: use Hahn-Banach theorem)