Doctoral Program — **Department of Mathematical Sciences**
Fall 2007 — Stevens Institute of Technology

General Examination: Morning Session

Problem 1: How many Sylow 3-subgroups does the symmetric group on 7 letters have?

Problem 2: Evaluate $\int_C e^{2/z} dz$ for the curve $C = \{z \in \mathbb{C} : |z| = 1\}$

Problem 3: Give examples:

(a) of a sequence of measurable sets $\{E_n\}$, $E_{n+1} \subset E_n$ for $n \geq 1$, that decrease to $\emptyset$, but $m(E_n) = \infty$ for all $n \geq 1$.

(b) of a sequence of integrable functions $\{f_n\}$ such that $f_n \to 0$ a.e., but $\int f_n \not\to 0$

(c) of a sequence of functions $f_n(x)$ such that $f_n \to 0$ in measure on $x \in [0, 1]$ but $\forall x \in [0, 1]$ $\lim_{n\to\infty} f_n(x)$ does not exist (i.e., f_n converges nowhere).

Problem 4: Let $p(x)$ be a polynomial on $\mathbb{R}$ with real coefficients. Show that if all its roots are real and distinct then:

$$[p'(x)]^2 > p(x)p''(x), \quad \forall x \in \mathbb{R}$$

Problem 5: Assume A is an $n \times n$ diagonalizable matrix with complex entries. Suppose $p(A)^2 = 0$ for some polynomial $p(\cdot)$ with complex coefficients. Show that in this case it is also true that $p(A) = 0$.

Problem 6: Let f be a complex valued function defined on the open disk of the complex plane ($\mathbb{D}$) such that the functions $g = f^2$ and $h = f^3$ are both analytic. Show that f is analytic on $\mathbb{D}$.