General Exam Part II

You have two hours to complete this part of the examination.
Name:

Good luck!

Problem 5: Let $(X, \mathcal{M}, \mu)$ be a measure space, $E_n \in \mathcal{M}$, $E_n \subseteq E_{n+1}$ for all $n = 1, 2, \ldots$, and $E = \bigcup_{n \geq 1} E_n$. Prove that $\mu(E_n) \underset{n \to \infty}{\longrightarrow} \mu(E)$.

Problem 6: Given a measure space $(X, \mathcal{M}, \mu)$, let f_n, $n = 1, 2, \dots$ and f be extended real-valued functions defined on X.

a. Define the notion of a measurable function on this space, the notions of convergence in measure, and almost sure convergence of the sequence $\{f_n\}$ to f.

b. If $f_n \underset{n\to\infty}{\longrightarrow} f$ almost surely, prove that $f_n \underset{n\to\infty}{\longrightarrow} f$ in measure.

Problem 7: Let $(X, \mathcal{M}, \mu)$ be a measure space. Given an extended real-valued function f defined on X, define $\int f d\mu$. Prove that $f(x) \geq 0$ for all $x \in E \subseteq \mathcal{M}$, and $\int_E f d\mu = 0$ imply that $f = 0$ almost everywhere on E.

Problem 8: State the Monotone and Dominated Convergence Theorems.