Ph.D. Program **Department of Mathematical Sciences**
Spring 2005 Stevens Institute of Technology

General Exam Part I

You have three hours to complete this part of the examination.
Name:

Good luck!

Problem 1: Prove that the mapping $x \to x^2$ is a homomorphism from any abelian group to itself. For which abelian groups is this map an isomorphism?

Problem 2: Let R be a commutative ring.

a. What is an ideal or R?

b. Suppose I is and ideal of R. Describe the quotient R/I. When is R/I a field? Prove your answer.

Problem 3: Let α be a complex number with $\mathrm{Re}\alpha > 0$. Evaluate

$$\int_{\mathbb{R}} e^{-\alpha x^2}\,dx.$$

Problem 4: Suppose the analytic function f has a domain $\Omega \subseteq \mathbb{C}$. Let u and v be, respectively, the real and the imaginary parts of f. Consider the level curve l_k of v corresponding to the value $k \in \mathbb{R}$. Prove that if l_k contains a segment of the straight line s, then $l_k = s \cap \Omega$.