Show all work. Answers without supporting work will not receive credit.

1. [8 pts] Evaluate the following limits:
   (a) \( \lim_{x \to 0} \frac{\ln(1 + x)}{x} \)
   (b) \( \lim_{x \to \infty} x(e^{\frac{1}{x}} - 1) \)

2. [8 pts] Evaluate \( \int_{0}^{\infty} xe^{-x} \, dx \)

3. [8 pts] Determine whether the following geometric series converge or diverge. If they converge, determine the sum of the series.
   a) \( 3 + \frac{6}{5} + \frac{12}{25} + \frac{24}{125} + ... \)
   b) \( \sum_{n=0}^{\infty} \frac{3^{n+1}}{2^{2n-2}} \)

4. [12 pts] Determine the interval of convergence for \( \sum_{n=1}^{\infty} \frac{(x + 3)^n}{3^{n-1} \sqrt{n}} \).

5. [10 pts] Derive the Taylor series expansion for \( f(x) = \sin\left(\frac{\pi x}{2}\right) \) about \( a = 1 \). Give your answer in compact form.

6. [8 pts]
   a) Using the Maclaurin series expansion for \( e^x \), find the Maclaurin series for \( f(x) = x^2 e^{-x} \). Give your answer in compact form.
   b) Find the sum of \( \sum_{n=0}^{\infty} \frac{3^n}{5^n n!} \)

7. [10 pts] The position vector of a particle is given by
   \( \vec{r}(t) = t^2 \hat{i} + 5t \hat{j} + (t^2 - 16t) \hat{k} \)
   a) Find the velocity and speed of the particle at the point \( P(1, 5, -15) \).
   b) When is the speed minimum?
8. [8 pts] Find the directional derivative of the function \( f(x, y, z) = x^2y + x\sqrt{1+z} \) at \((1, 2, 3)\) in the direction \( \vec{v} = 2\hat{i} + \hat{j} - 2\hat{k} \).

9. [6 pts] Determine whether the given pairs of vectors are orthogonal, parallel, or neither.
   a) \( 2\hat{i} + 5\hat{j} + 6\hat{k}, \ 6\hat{i} + 15\hat{j} + 18\hat{k} \)
   b) \( \hat{i} + \hat{j} + \hat{k}, \ 2\hat{i} + 3\hat{j} + 5\hat{k} \)
   c) \( 2\hat{i} + 3\hat{j} + 5\hat{k}, \ 2\hat{i} - 2\hat{j} \)

10. [6 pts] Find the equation of the line that passes through \((2,1,0)\) and perpendicular to both \( \hat{i} + \hat{j} \) and \( \hat{j} + \hat{k} \).

11. [6 pts] Find the equation of the plane through \((2,1,0)\) and parallel to the plane \( x + 4y - 3z = 1 \).

12. [10 pts] Find the maximum and minimum value of \( f(x, y) = x^2y \) subject to the constraint \( x^2 + y^2 = 1 \).

13. [10 pts] Evaluate \( \iint_R y \, dA \) where \( R \) is the region above the \( x \)-axis between the circles (with center 0) of radius 1 and radius 2.

14. [10 pts] Consider the volume of the solid that lies under the surface \( z = x^2 + y^2 \) in the first octant, above the \( xy \)-plane, and inside the cylinder \( x^2 + y^2 = 2x \).
   a) Using Cartesian coordinates set up the double integral for finding the volume in two different ways: \( \iint_R f(x, y) \, dy \, dx \) and \( \iint_R f(x, y) \, dx \, dy \). DO NOT EVALUATE THE TWO INTEGRALS.
   b) Using polar coordinates set up the double integral and evaluate it to find the volume.