1. [10 pts] Evaluate the improper integral if it is convergent, or show that it is divergent.

(a) \( \int_{0}^{\infty} xe^{-x^2} \, dx \)
(b) \( \int_{-\infty}^{\infty} \frac{x}{1 + x^2} \, dx \)

2. [10 pts] Find the volume under the paraboloid \( z = x^2 + y^2 \) and above the region \( R \) in the plane \( z = 0 \), where \( R = \{(x, y) \mid -2 \leq x \leq 2, \ 0 \leq y \leq 1\} \).

3. [10 pts]

(a) Find the angle between the vectors \( \langle 4, 2, 2 \rangle \) and \( \langle 1, 2, -1 \rangle \).

(b) For what values of \( b \) are the vectors \( \langle -6, b, 2 \rangle \) and \( \langle 1, -2b, 4b \rangle \) orthogonal?

4. [10 pts] Find the Taylor series and its interval of convergence for \( f(x) = \ln x \), centered at \( a = 3 \).

5. [10 pts] For the function \( F(x, y, z) = x^2y + 2x(1 + z) \), determine the following:

(a) the gradient of \( F \) at the point \( P(1, -1, 3) \);

(b) the rate of change of \( F \) in the direction of vector \( \vec{a} = 2\vec{i} + \vec{j} - 2\vec{k} \);

(c) the equation of the tangent plane to the level surface \( F(x, y, z) = 7 \), at the point \( P(1, -1, 3) \).

6. [10 pts] Determine whether the series is convergent or divergent:

(a) \( \sum_{n=1}^{\infty} \frac{5n^4 + 1}{3n^4 - n + 2} \)
(b) \( \sum_{n=1}^{\infty} \frac{5n^2 + 1}{3n^4 - n + 2} \sin(n) \)
7. [10pts] Find the limits:  
(a) \( \lim_{x \to 0} \frac{\ln(1 + x)}{x} \)  
(b) \( \lim_{x \to \infty} x \left( e^{1/x} - 1 \right) \)

8. [10pts]  
(a) Verify that \( u(t, x) = \cos(2x - t + 3) \) is a solution to the equation, \( 2u_t + u_x = 0 \).

(b) Suppose \( z = xy + f(u(x, y)) \), where \( f \) is a differentiable function of \( u \).
If \( u(x, y) = x^2 + y \), show that \( z \) satisfies the differential equation,  
\[
\frac{\partial z}{\partial x} - 2x \frac{\partial z}{\partial y} = y - 2x^2.
\]

9. [10pts] Find the absolute maximum and minimum values of \( f(x, y) = 3 + xy - x - 2y \) on the triangular region with vertices \((0, 0), (4, 0), \) and \((2, 3)\).

10. [10pts] For \( t \geq 0 \), a particle’s position is given by the function,  
\[
\vec{r}(t) = 2 \cos(\pi t) \vec{i} + \sin(\pi t) \vec{j} + \left(1 - \frac{1}{t + 1}\right) \vec{k}
\]
(a) Determine the particle’s velocity and speed at the point \( P(-2, 0, 1/2) \).

(b) Find the equation of the tangent line to the curve at \( P(-2, 0, 1/2) \).

11. [10pts]  
(a) Determine the sum \( \sum_{n=0}^{\infty} \frac{(-2)^{3n-2}}{10^n} \), if it exists.

(b) Determine the power series and its interval of convergence for \( f(x) = \frac{x^2}{x + 1} \), centered at \( x = 0 \).

12. [10pts] Evaluate the double integral \( \int \int_{D} xy \, dA \) where \( D \) is the triangular region with vertices \((0, 0), (2, 1), \) and \((3, 0)\).