PLEDGE

I pledge my honor that I have abided by the Stevens Honor System.
The above pledge is to be written and signed by the student in the
space below after finishing the examination.

Stevens Institute of Technology
Ma 116 – Final Exam May 9, 2001

Name: ID: ____________________________

Check your lecture: □ Miller □ Gilman □ Tabanjeh (11:00) □ Tabanjeh (12:00)

Closed book and closed notes. Calculators and laptops are not allowed.
Show all of your work. Answers without supporting work may not receive credit.

Pledge and Sign:

Problem I. Determine whether or not the following series converge. Sum the series if possible.

a. (5 pts) $4 + 4/3 + 4/9 + 4/27 + 4/81 + \cdots$

b. (5 pts) $\sum_{n=1}^{\infty} \frac{n}{n + 17}$

c. (5 pts) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^n}{3^n}$

Problem II.

a. (7 pts) Find the radius of convergence for the series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n4^n}$.

b. (8 pts) Use a power series expansion for $\ln x$ to find $\lim_{x \to 1} \left( \frac{\ln x}{x-1} \right)$.

Problem III.

a. (5 pts) Find a unit vector in the direction of $3\vec{i} - \vec{j} + 6\vec{k}$.

b. (5 pts) Compute $\vec{v} \times \vec{w}$ if $\vec{v} = 3\vec{i} - 4\vec{j}$ and $\vec{w} = 4\vec{i} - \vec{j} + 2\vec{k}$.

c. (5 pts) Find an equation for the plane through the point $(2, 0, 1)$ and perpendicular to the vector $4\vec{i} + \vec{j} - 3\vec{k}$.
Problem IV. For each of the following exercises, let \( g(x, y) = 4x^2e^{x+y} \). 

a. (5 pts) Compute \( \frac{\partial^2 g(x, y)}{\partial x \partial y} \).

b. (5 pts) Compute \( \frac{dg}{dt} \) at \( t = 1 \) if \( x = 2t, \ y(1) = 3 \), and \( \frac{dy}{dt} = -2 \) when \( t = 1 \).

c. (5 pts) Compute the rate of increase of \( g \) at the point \( (x, y) = (2, 1) \) in the direction of the vector \( \vec{v} = \vec{i} - 2\vec{j} \).

Problem V.

a. (6 pts) Sketch the level curves of \( f(x, y) = x + y \), using the grid provided.

b. (9 pts) Find and classify the critical points of \( f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2 \).

Problem VI.

a. (7 pts) Consider the hexagon inscribed within the unit circle \( x^2 + y^2 = 1 \) as shown in the upper figure at the right. When this hexagon is rotated around the \( y \)-axis, it generates a solid of revolution consisting of a cylinder and two cones, as shown in the lower figure. Determine the radius \( R \) and cylinder height \( H \) that maximize the volume of the solid. Use the Lagrange multiplier method to set up the algebraic equations for solving this optimization problem (but do not solve the equations).

b. (8 pts) Find the shortest distance between the origin and the surface \( z^2 = x^2y + 4 \).

Note: In the final calculations one or more of the following approximations should be helpful: \( 2^{1/3} \approx 1.26, \ 2^{2/3} \approx 1.59, \ 2^{4/3} \approx 2.52, \ 2^{5/3} \approx 3.17. \)
Problem VII.

a. (7 pts) Evaluate \( \int \int_{R} xy \, dA \) where \( R \) is the region bounded by the parabola \( y = x^2 \) and the line \( x + y = 2 \).

b. (8 pts) Use polar coordinates to convert \( \int \int_{S} (x^2 + y^2) \, dA \) to an iterated integral where \( S \) is the region in the plane that lies above the line \( y = 1 \) and below the circle \( x^2 + y^2 = 4 \). (Set up the iterated integral in polar coordinates but do not evaluate.)

Problem VIII.

a. (5 pts) A particle starts at the origin at time \( t = 0 \) and moves with the velocity function \( \mathbf{v}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + t^2 \mathbf{k} \). Find the position of the particle at time \( t = 1 \).

b. (5 pts) Write down the Maclaurin series for the function \( e^x \) \( (e^x = \sum_{n=0}^{\infty} c_n x^n) \) and use this to estimate the constant \( e^{-1} \) by summing the Maclaurin series through \( n = 4 \).

c. (5 pts) Use the Alternating Series Estimation Theorem to find an upper bound on the error in your estimate from the previous question.