EasyUC: Using EasyCrypt to Mechanize Proofs of Universally Composable Security

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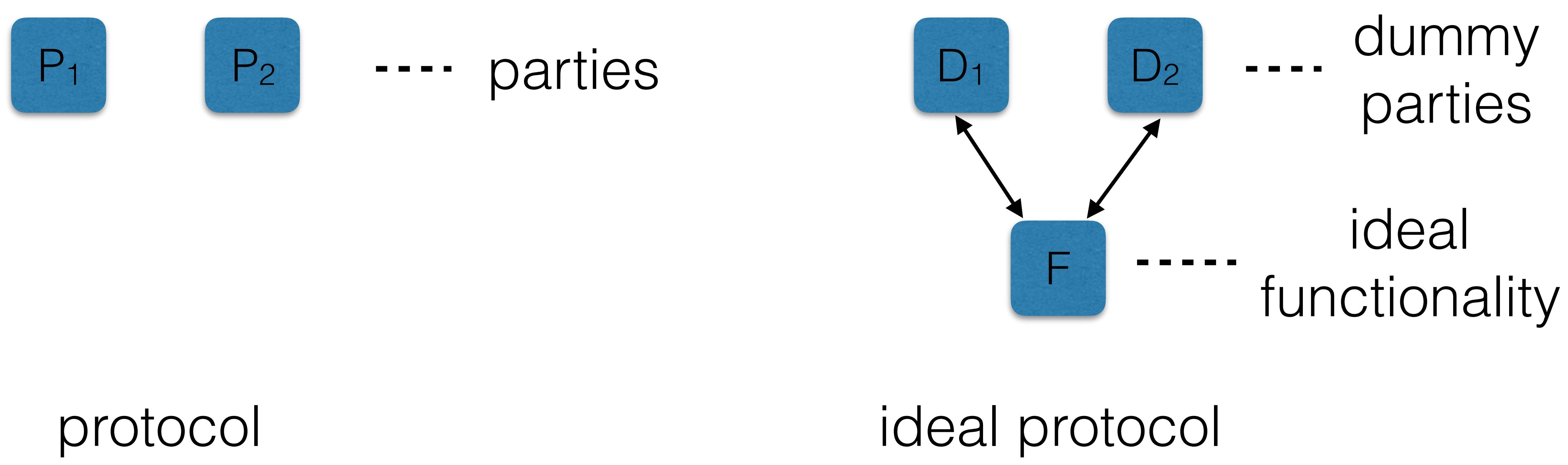
Universally Composable Security

- Universally Composable (UC) Security (Canetti, …) is a refinement of the real/ideal paradigm supporting modular proof development
- In UC, a protocol interacts with
  - an *environment*, which supplies protocol inputs and consumes protocol outputs, and
  - an *adversary*, which is given certain powers to observe or corrupt the protocol
- The environment and adversary (may) communicate
- UC uses a *coroutine style* of message passing in which control is transferred along with data

Universally Composable Security

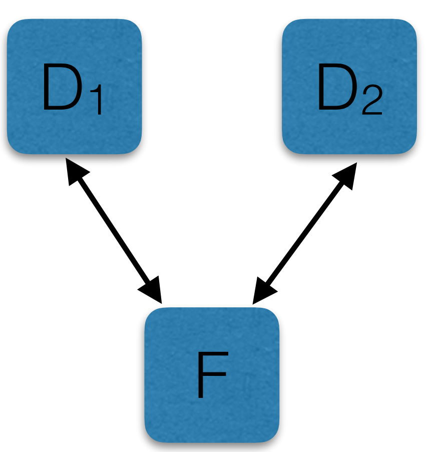
- A *protocol* consists of some number of protocol parties
- An *ideal protocol* consists of an *ideal functionality* combined with *dummy parties* transferring inputs/outputs to/from the ideal functionality
  - Specifies desired functionality, plus leakage to simulator



Universally Composable Security

- Because we always work with an ideal functionality and its dummy parties *as a unit*, and we needed a *neutral term* for a protocol or an ideal protocol, we settled on

protocol



ideal protocol

Universally Composable Security

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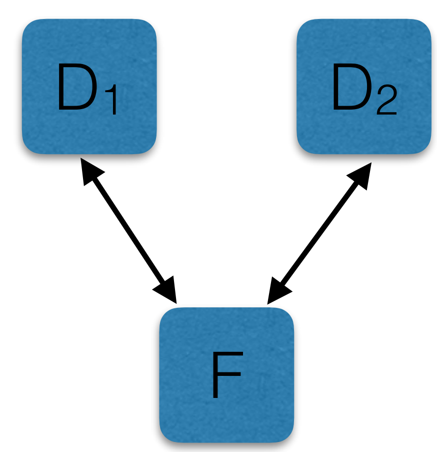
  - calling ideal protocols *ideal functionalities*, and
  - calling protocols real functionalities
- Thus a *functionality* is either a real or ideal functionality

real functionality



ideal functionality

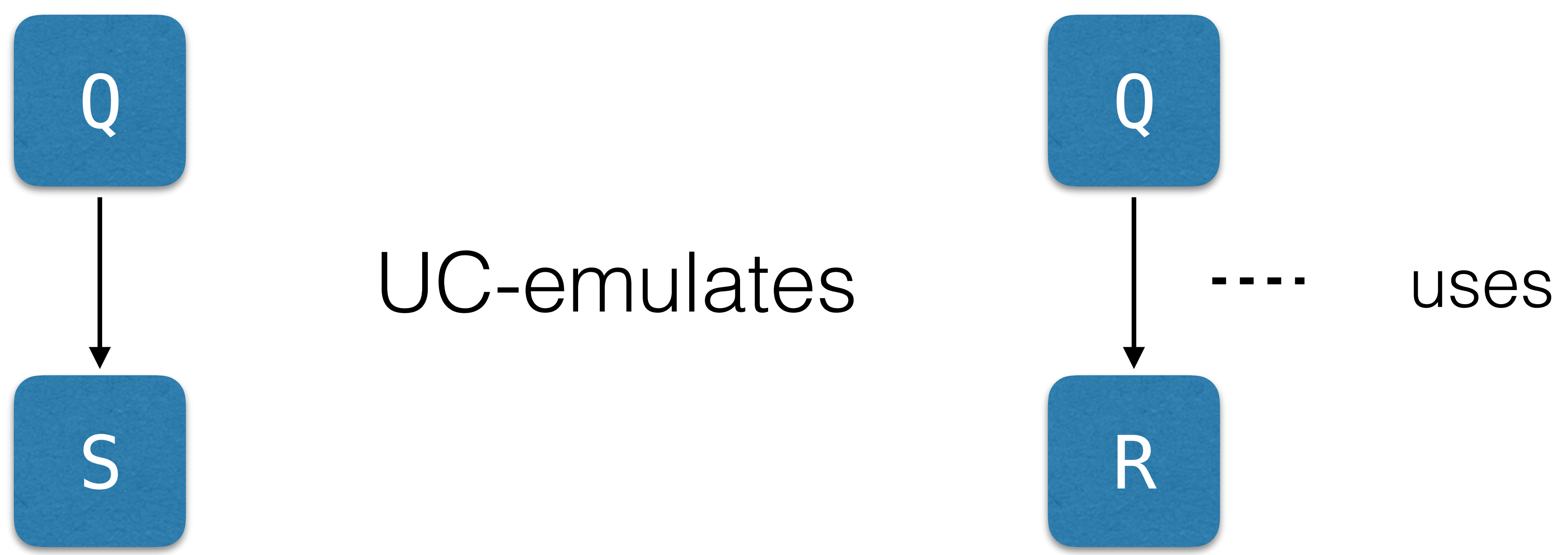
Universal Composable Security

- A real functionality `RF` *UC-emulates* an ideal functionality `IF` iff, there is an efficient, black box simulator `Sim`, such that, for all efficient adversaries `Adv`, and for all efficient environments `Env`, `Env` can't tell if it is interacting with
  - `RF/Adv` (the real game), or
  - `IF/Sim(Adv)` (the ideal game)
- More precisely, the environment yields a boolean judgment, and we want the absolute value of the difference between the probabilities of the environment returning true in the real and ideal games to be small
- This definition is the same when the second functionality is also a real functionality
- UC-emulation is trivially transitive: the simulators compose

Universal Composable Security

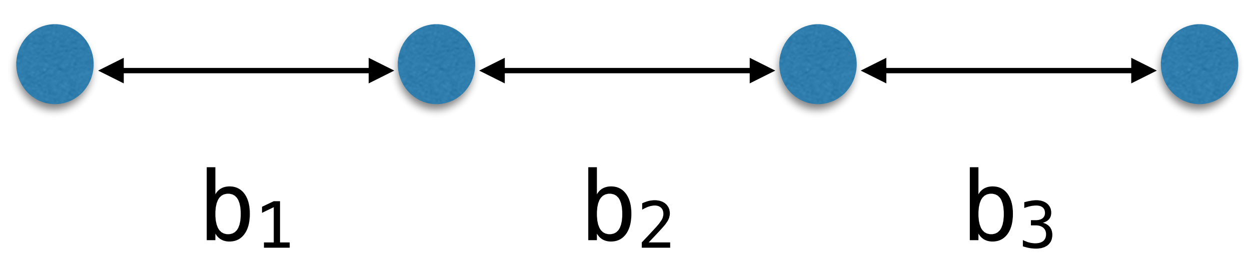
- The UC Composition Theorem says that:
  - if **S** UC-emulates **R**, and **Q** is a functionality using **R**,

  then if we change **Q** to use **S** instead of **R** (this is the UC composition operator), the result will UC-emulate **Q**



Sequence of Games Approach

- In general, it takes some number of steps to connect real and ideal games
  - Each step establishes an upper bound on the ability of the environment to discriminate between the two games
  - The sum of these upper bounds is an upper bound on the ability of the environment to discriminate between the real and ideal games
  - Steps may be proved by reductions, up-to bad reasoning, code motion, …



Proof Mechanization

- Several frameworks have been developed for mechanizing cryptographic security proofs in the sequence of games approach:
  - CryptoVerif (Blanchet) is semi-automated, guided by hints
  - FCF (Petcher & Morrisett) is embedded in Coq
  - CryptHOL (Basin, Lochbihler & Sefidgar) is embedded in Isabelle/HOL
  - EasyCrypt (Barthe, Grégoire, Strub, …, Stoughton, …) is a standalone proof assistant, with a fairly small and well-studied TCB
- We're using EasyCrypt partly because it directly handles modules — including abstract ones like adversaries — with their own local, private state

EasyCrypt's Modules

- Modules consist of global variables and procedures
- Modules may be parameterized, e.g., by adversaries or environments
- Procedures are written in a simple imperative language, with while loops and random assignments (choosing values from probability sub-distributions)

EasyCrypt's Logics

- EasyCrypt has four logics:
  - a Probabilistic Relational Hoare Logic (pRHL) for proving relations between pairs of games
  - a Probabilisitic Hoare logic (pHL) for proving probabilistic facts about single games
  - an ordinary Hoare logic (HL)
  - an ambient higher-order logic for proving mathematical facts and connecting judgements from the other logics

EasyCrypt's Proofs and Theories

- Proofs are structured as sequences of lemmas
- Lemmas are proved using tactics, as in Coq
- EasyCrypt theories may be used to group definitions, modules and lemmas together
- Theories may be specialized via cloning

UC in EasyCrypt

- We are in the early stages of researching how UC security may be mechanized in EasyCrypt
- A major challenge is how to deal with UC's coroutine style of communication in EasyCrypt's procedural programming language
- Our approach is to give functionalities, the adversary and parts of the environment *addresses* (lists of integers), and to build abstractions that route *messages* to their destinations
    - The empty list, `[]`, is the root address of the environment

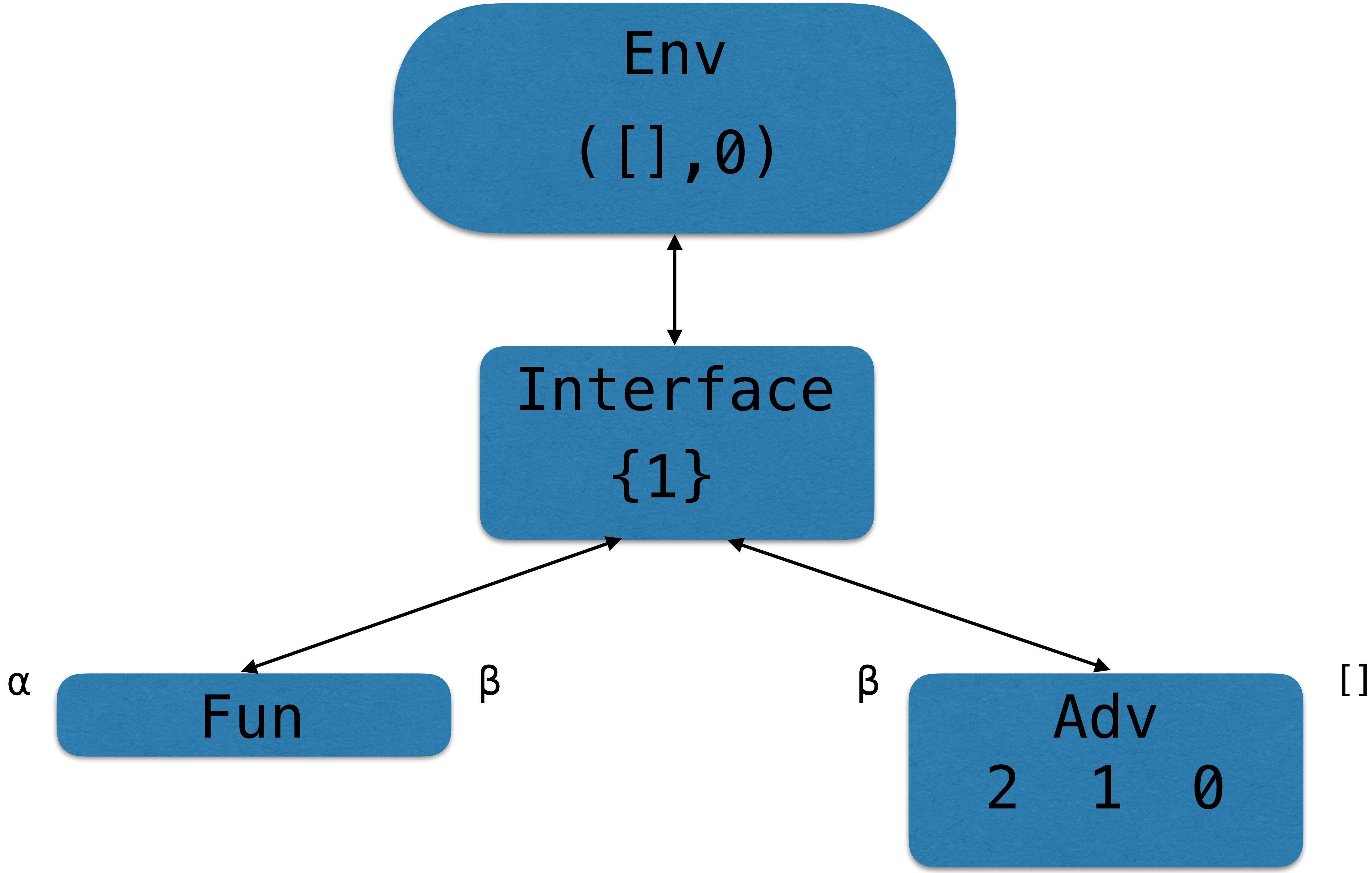
UC in EasyCrypt

- On top of the addressing system, we have a simple naming scheme based on *ports* (α, i), where α is an address, and i is an integer (a *port index*)
  - `([], 0)` is the environment's default port
  - Each of a functionality's parties has some number of ports
- Messages can be
  - "*direct*" — providing functionality inputs or reporting functionality outputs; or
  - "*adversarial*" — communication between environment and adversary, or functionality and adversary

Functionalities in EasyCrypt

- We realize functionalities as modules
  - The parties of a functionality live within a single module
- Functionalities may have sub-functionalities, with sub-addresses
  - A parent functionality can choose which messages from the environment to forward to its sub-functionalities
- Modules in EasyCrypt may be parameterized, allowing the UC composition operator to be realized as module application
- Multiple instances of functionalities can be statically created using EasyCrypt's cloning mechanism

Interface Firewall



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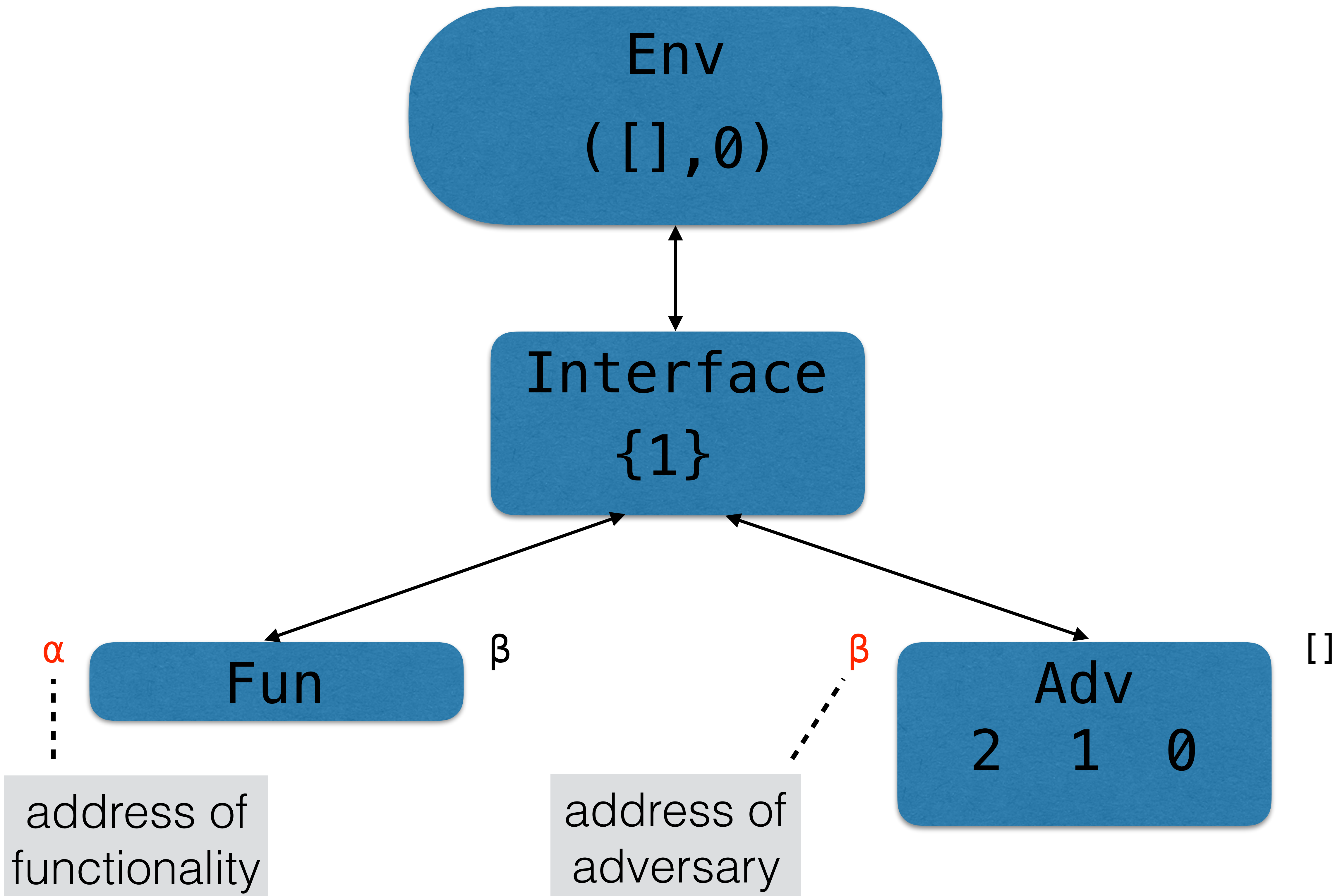

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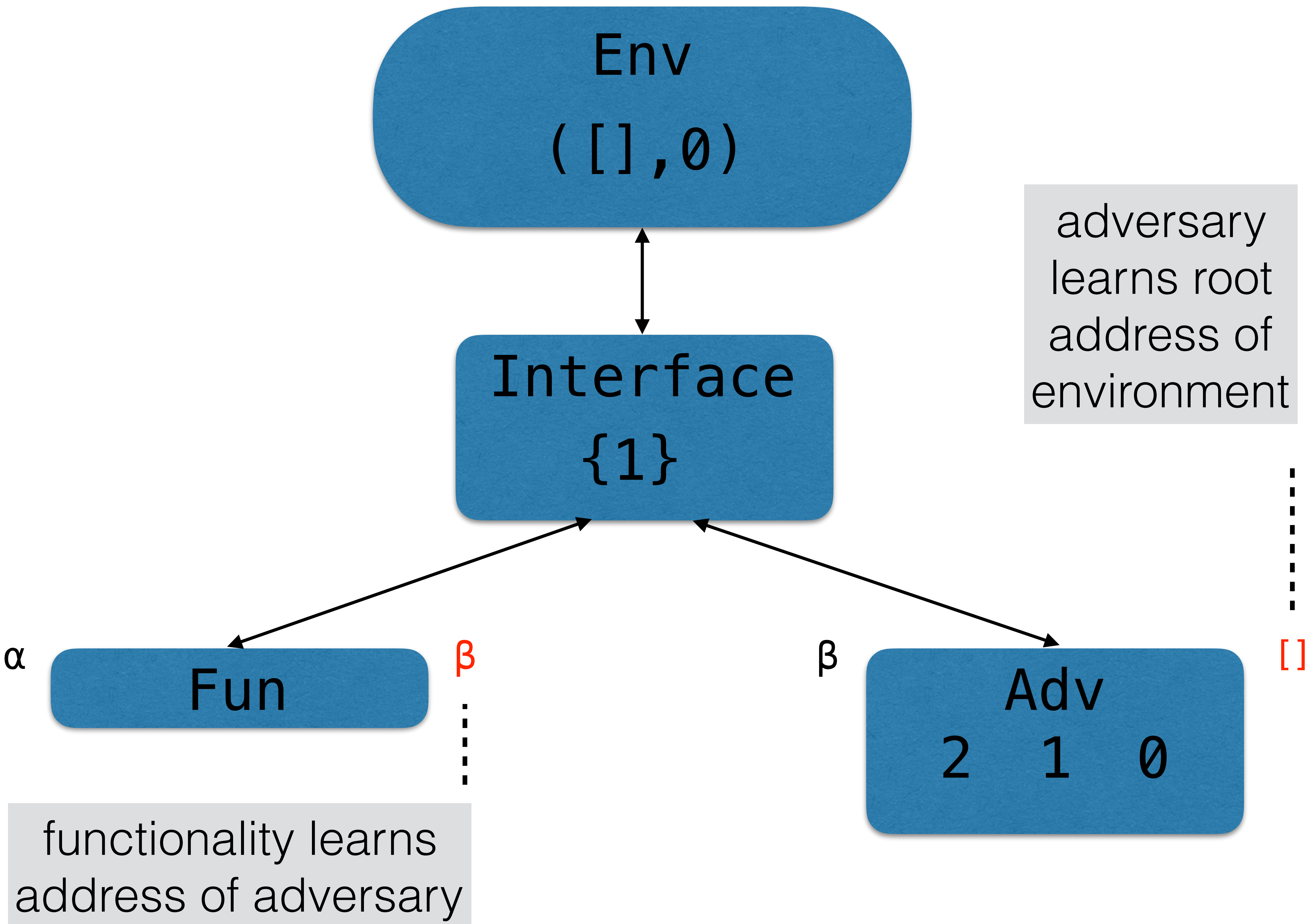

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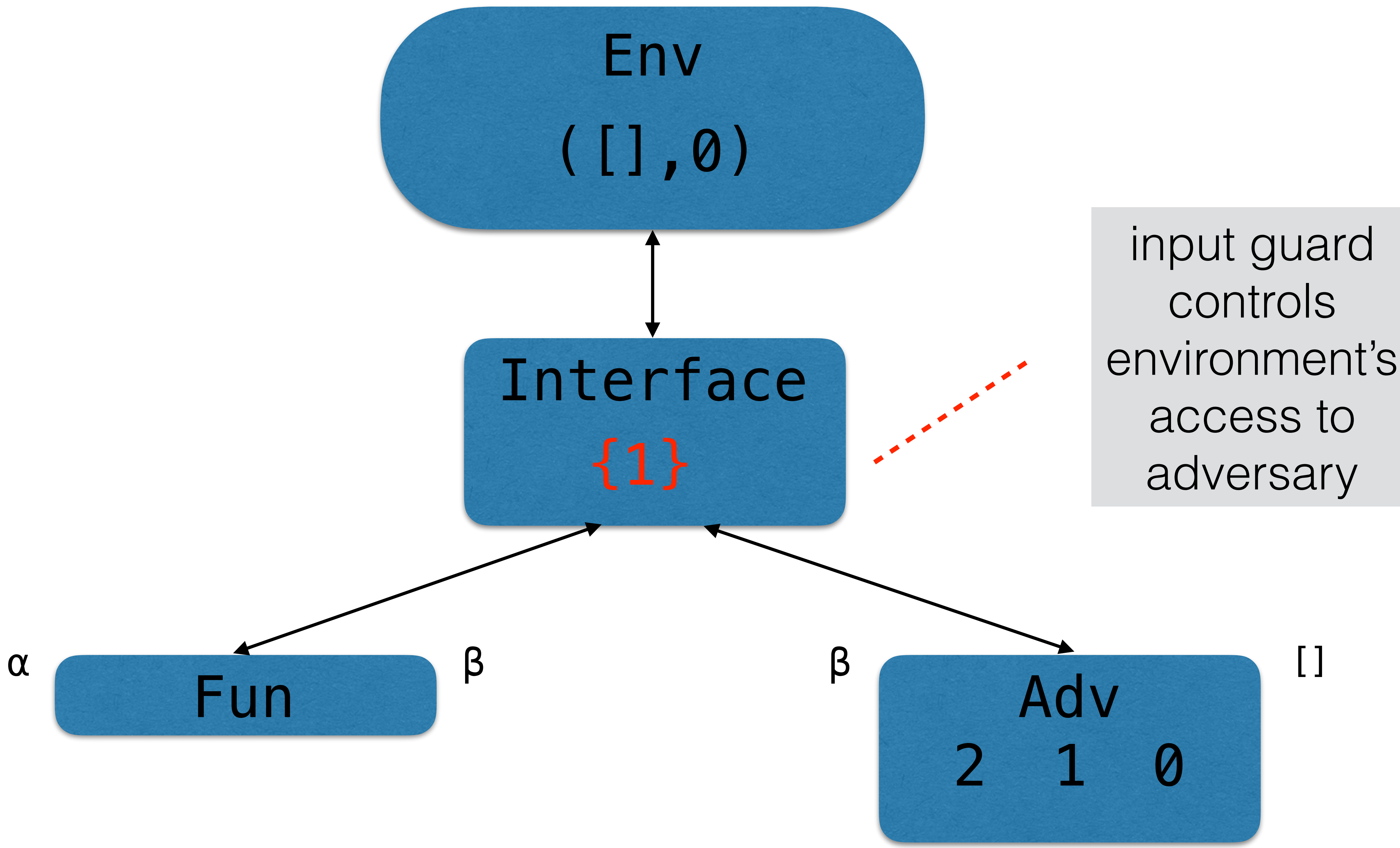

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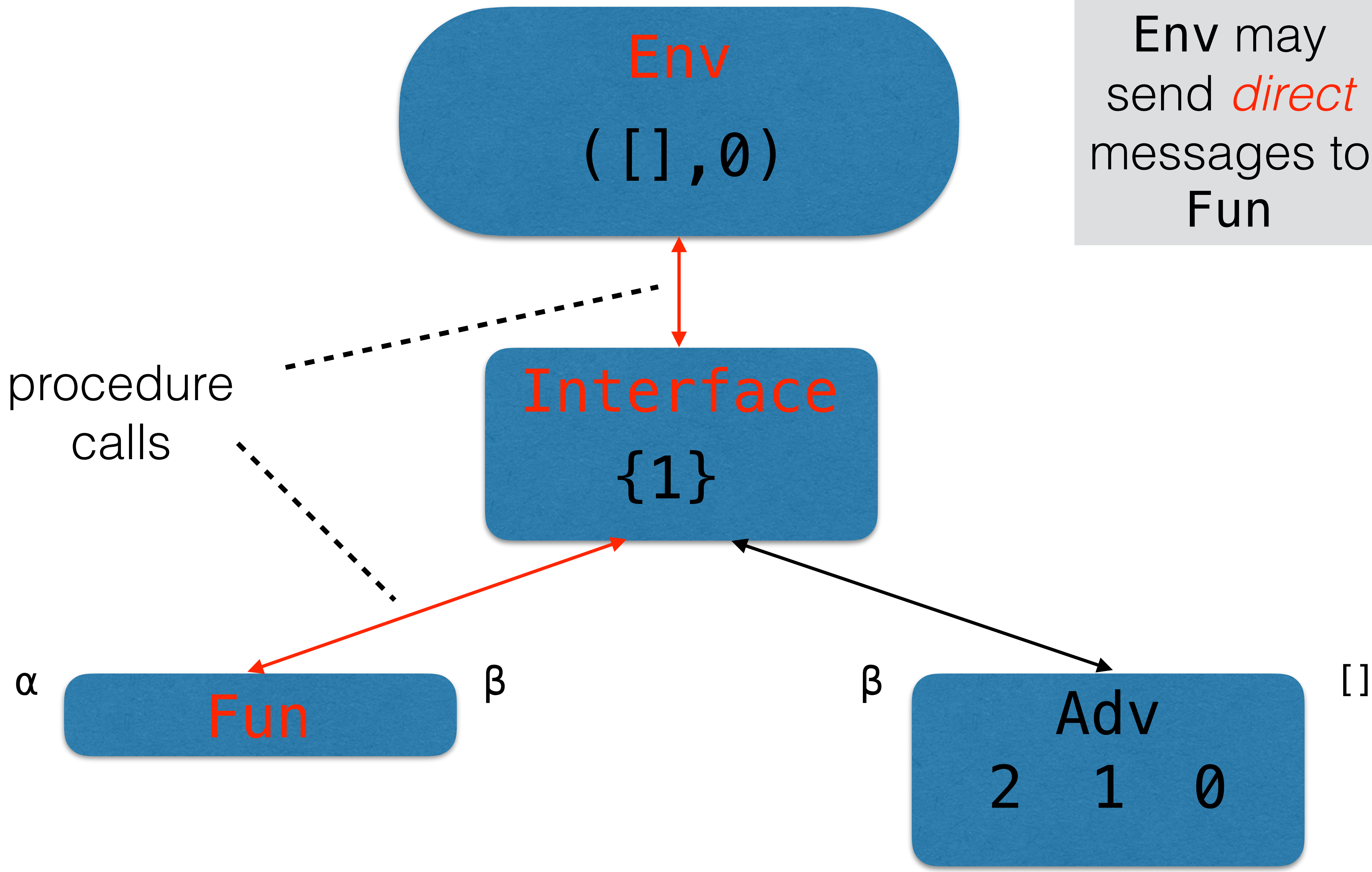
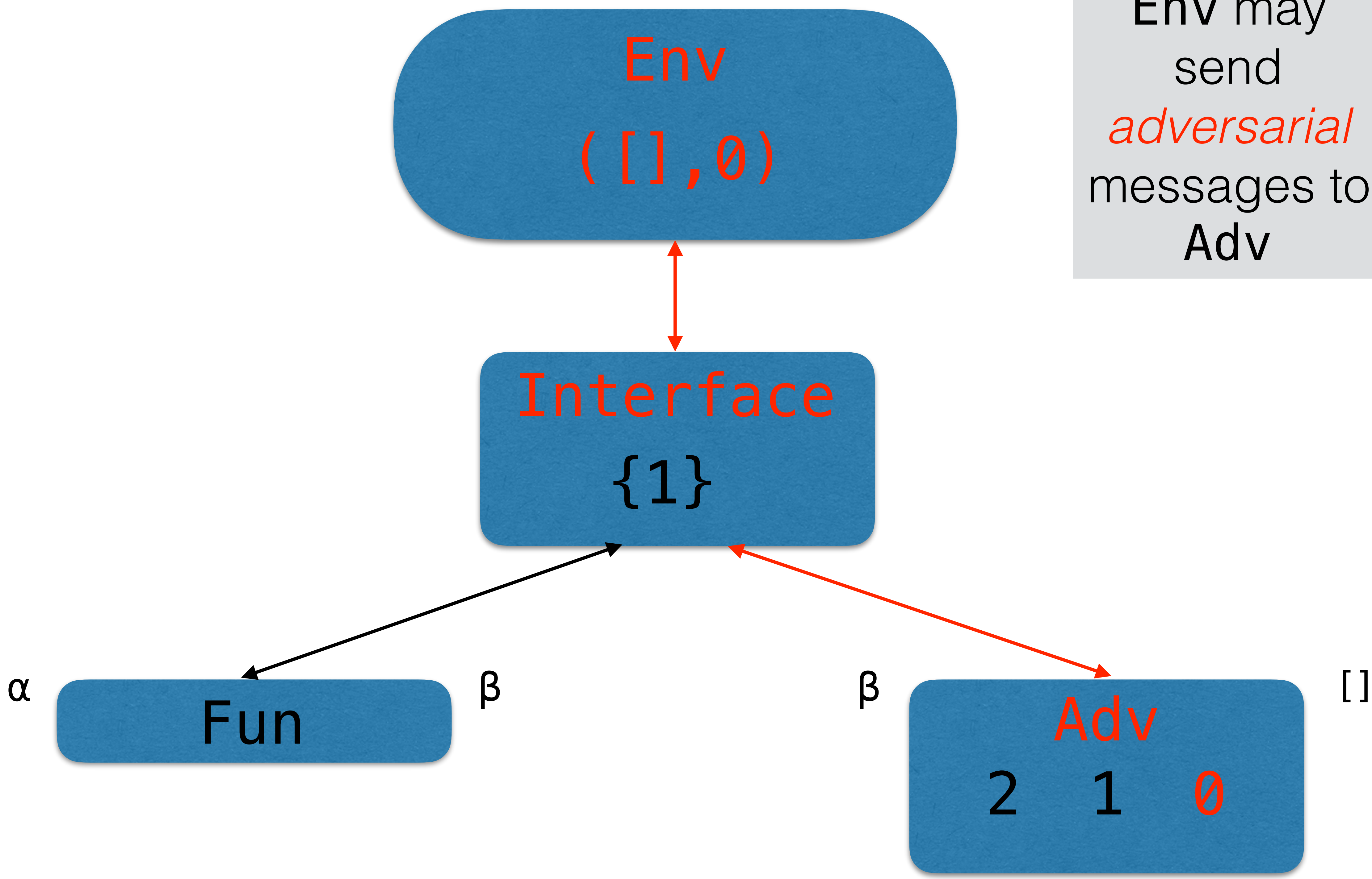


Env may send *direct* messages to **Fun**

Interface Firewall

Env may send *adversarial* messages to **Adv**



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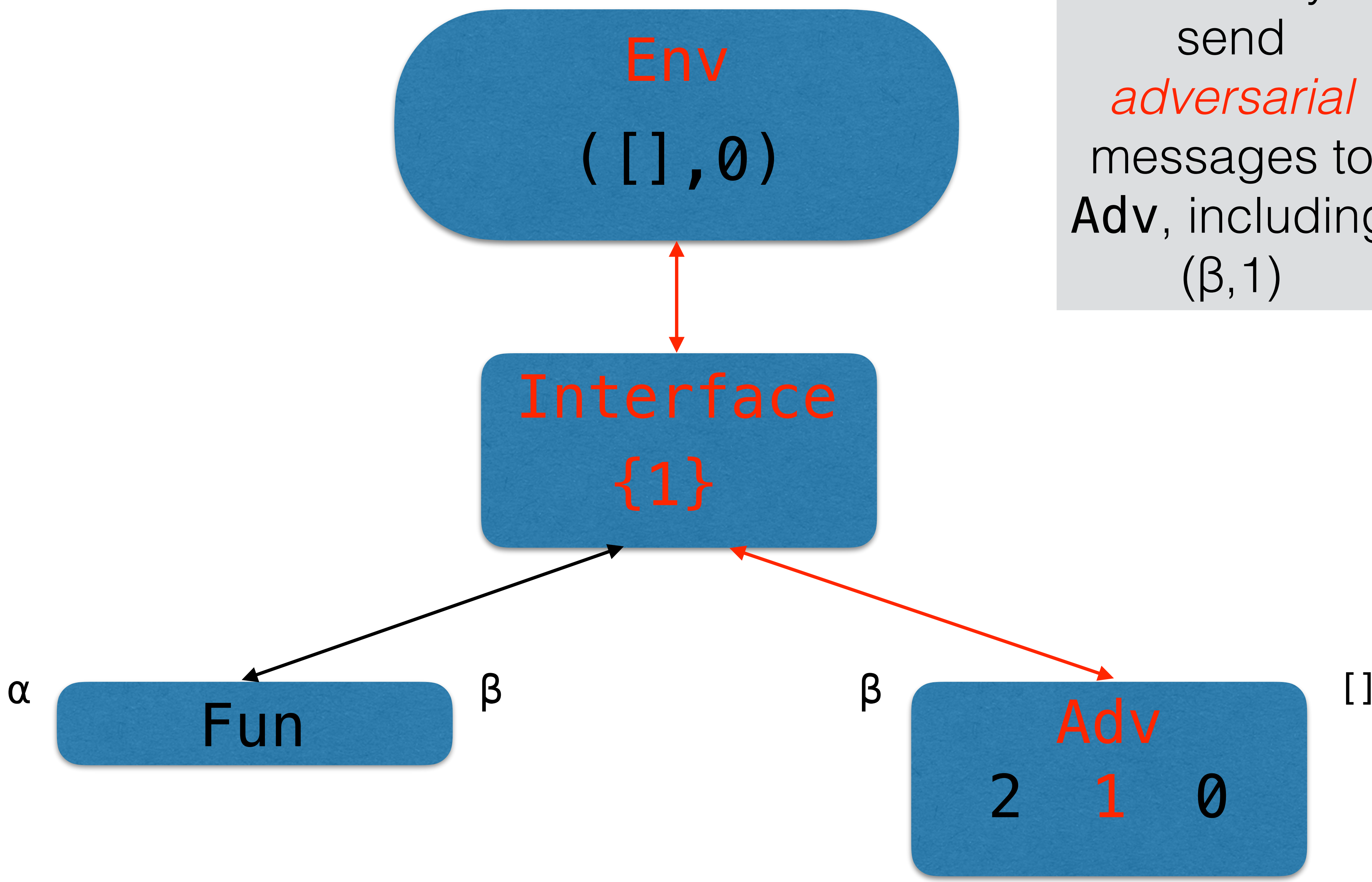
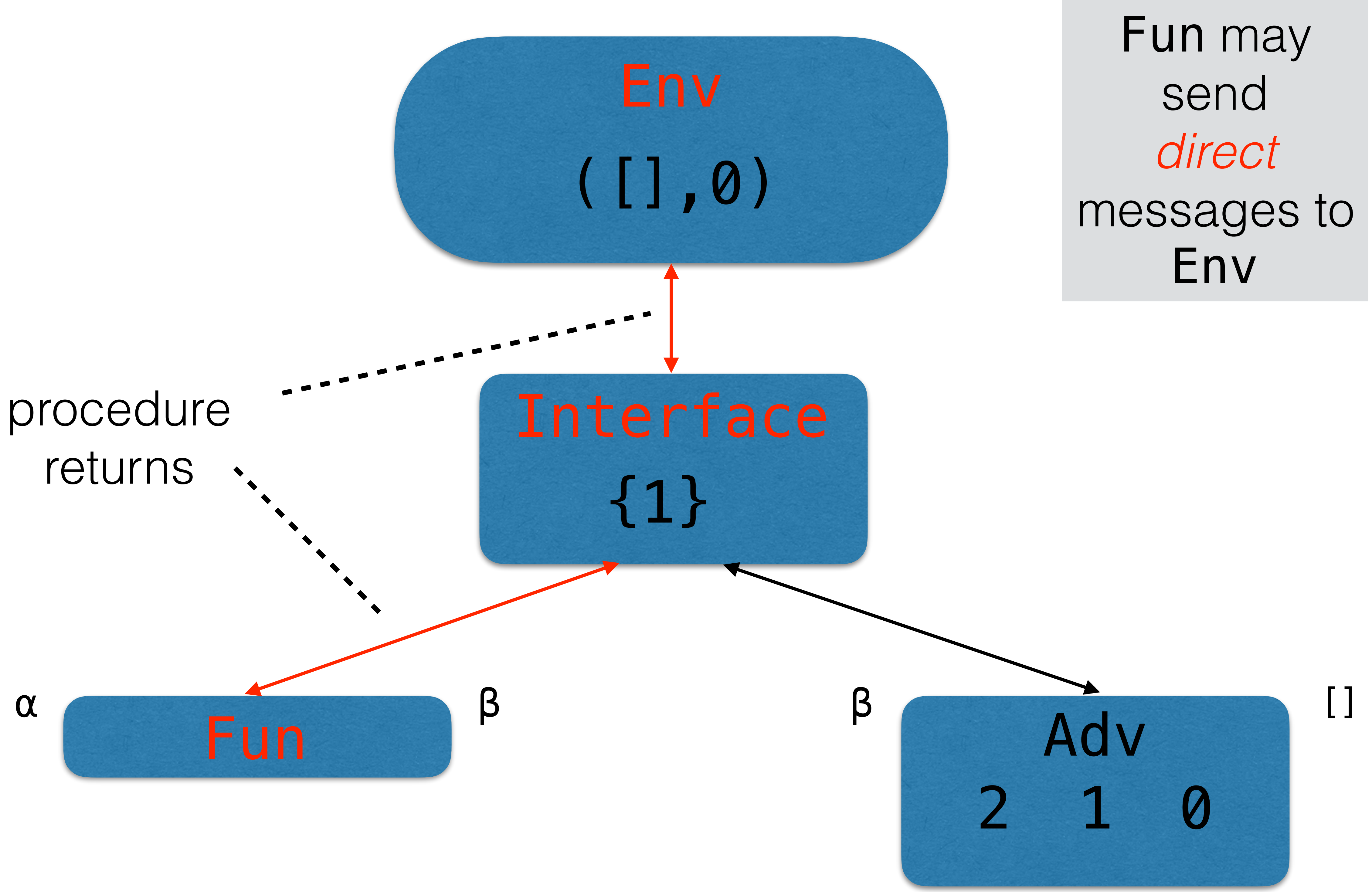


Env may send *adversarial* messages to **Adv**, including (β,1)

Interface Firewall



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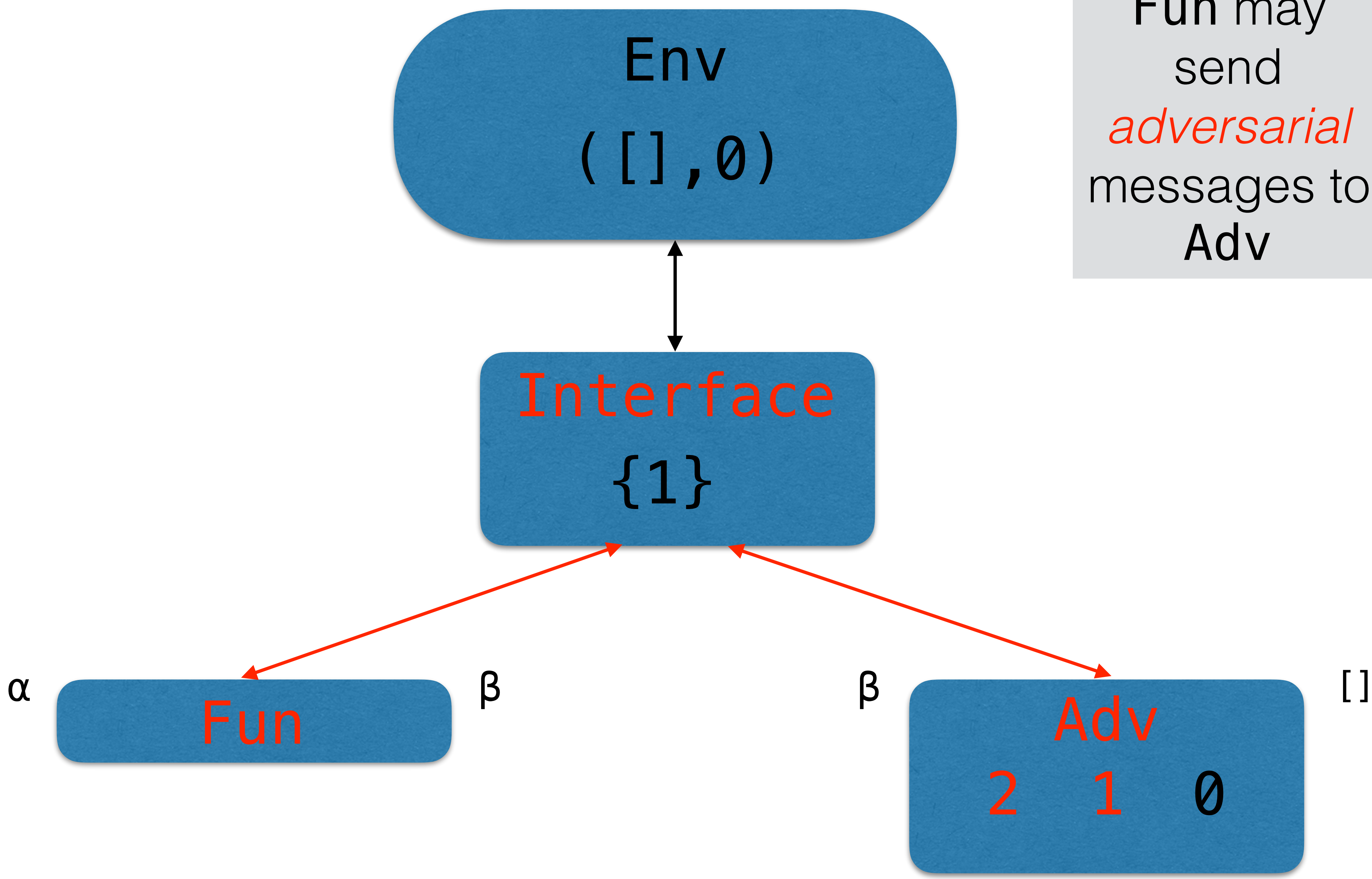

Fun may send *adversarial* messages to **Adv**

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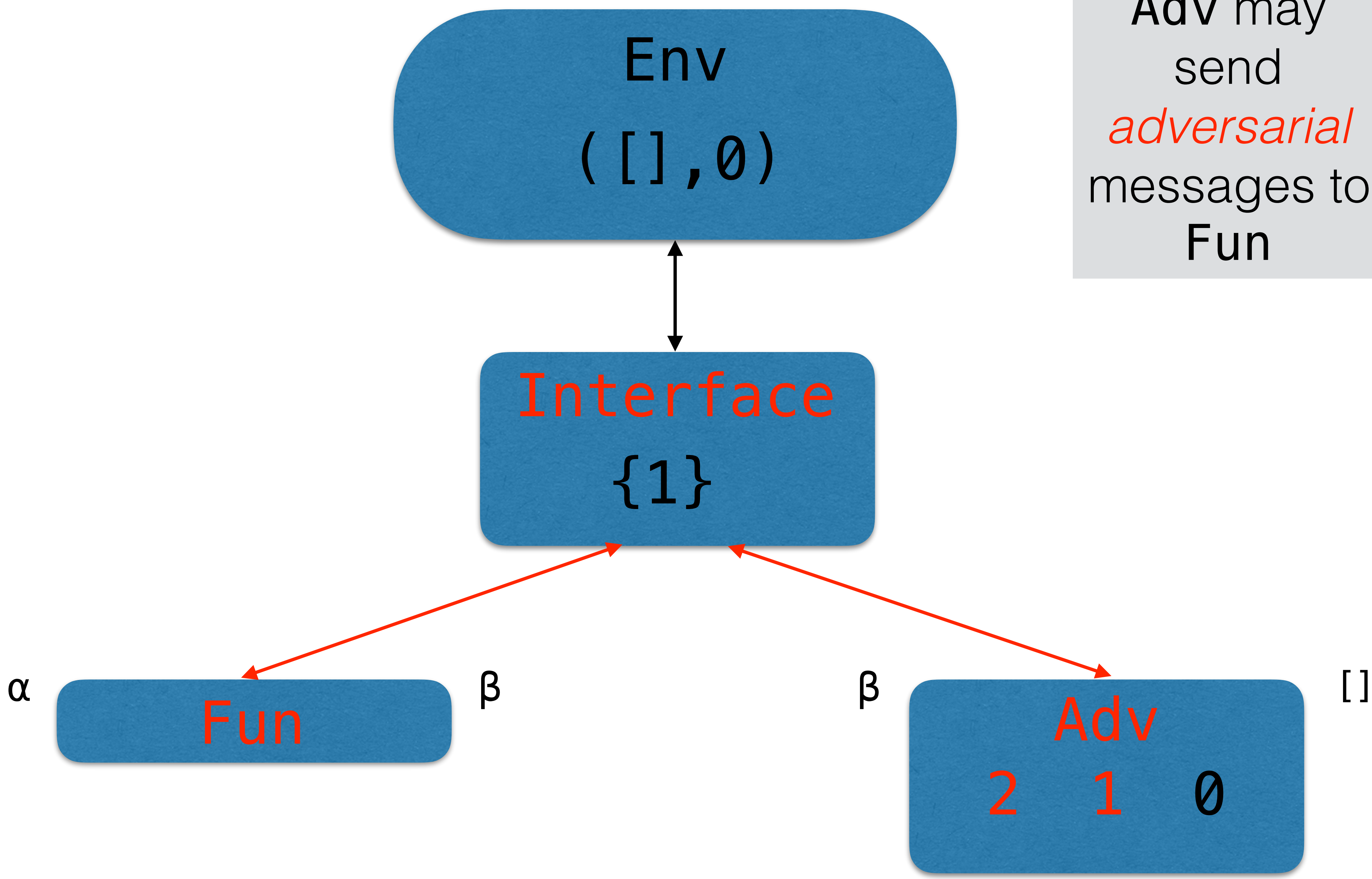

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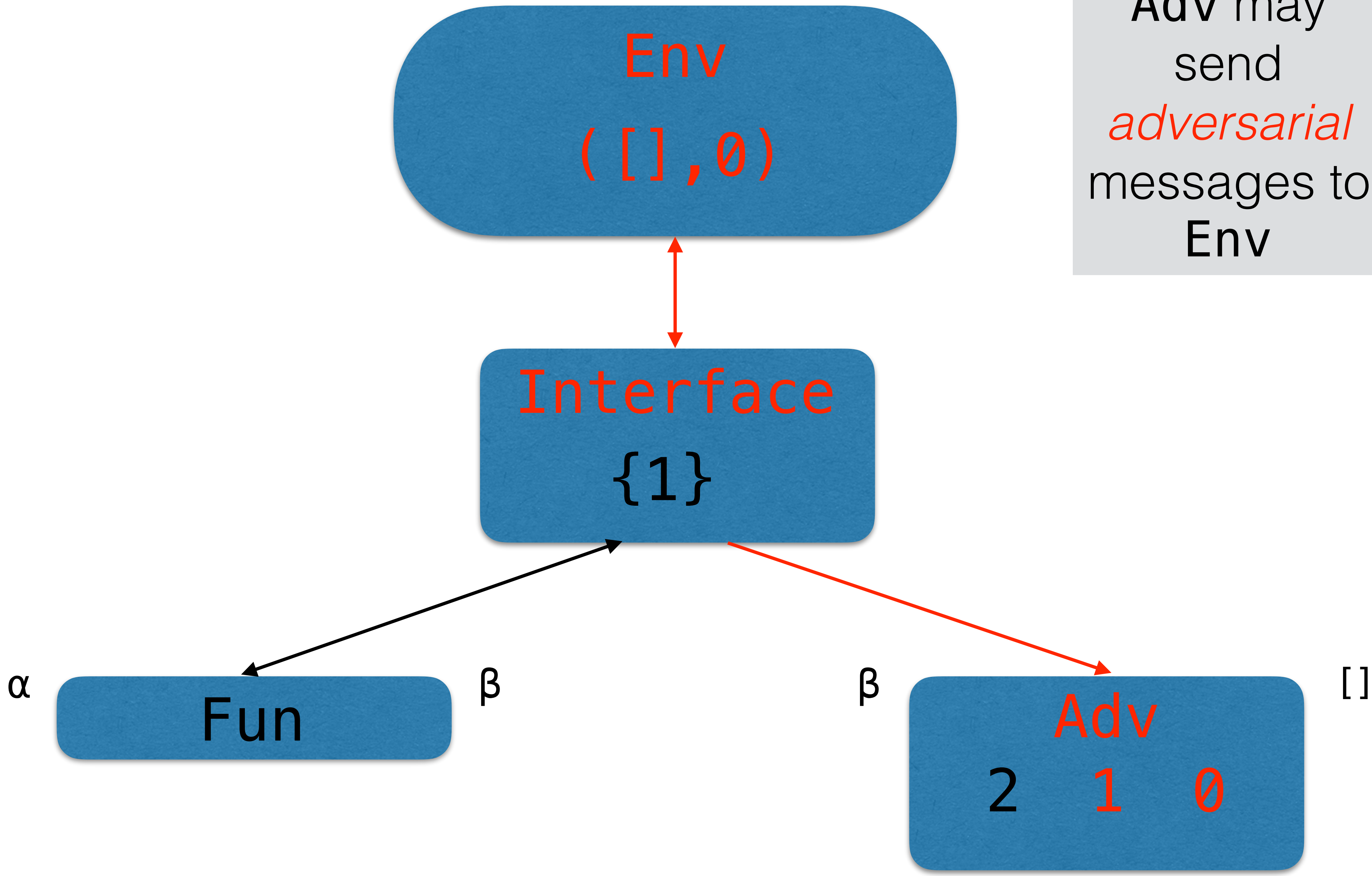
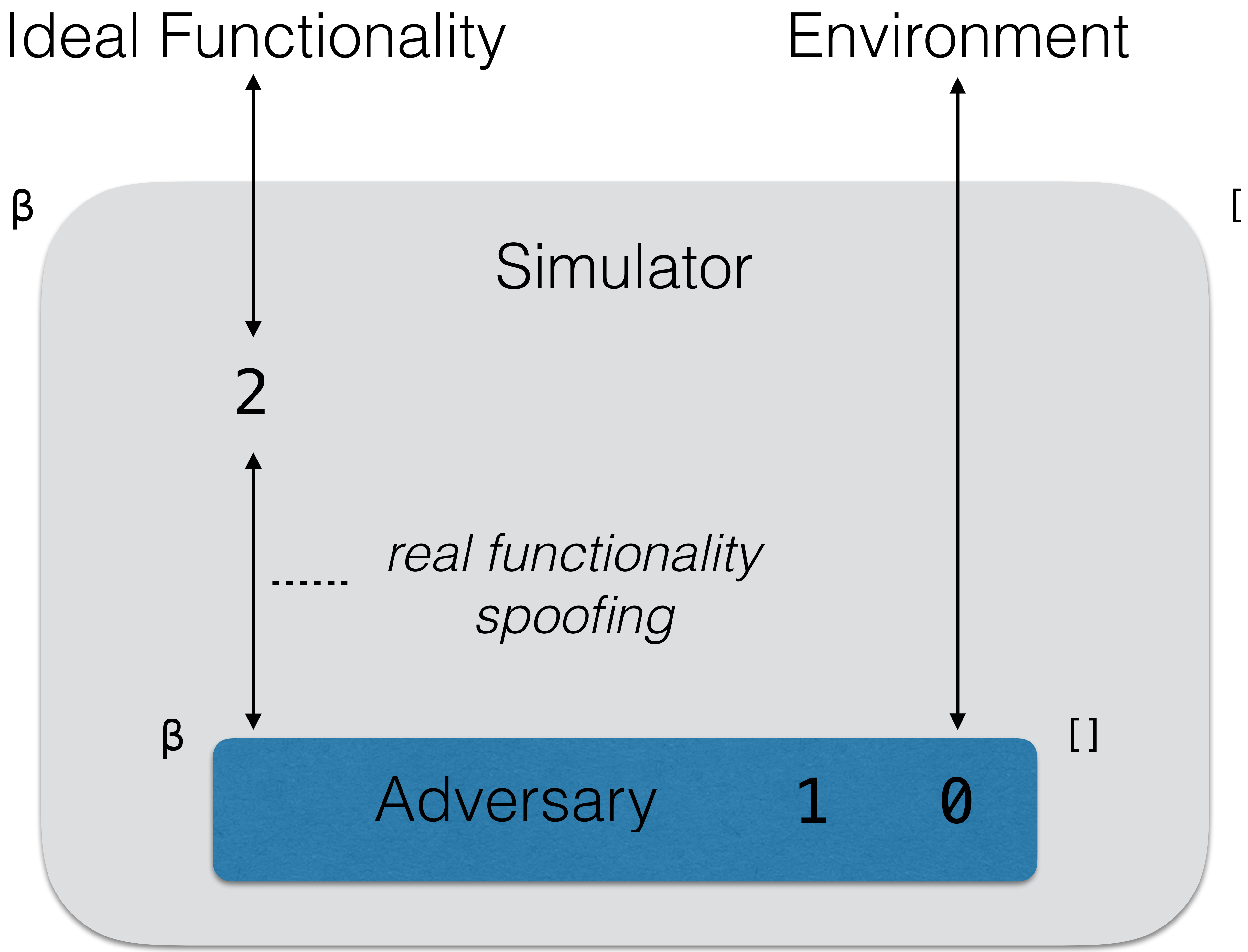
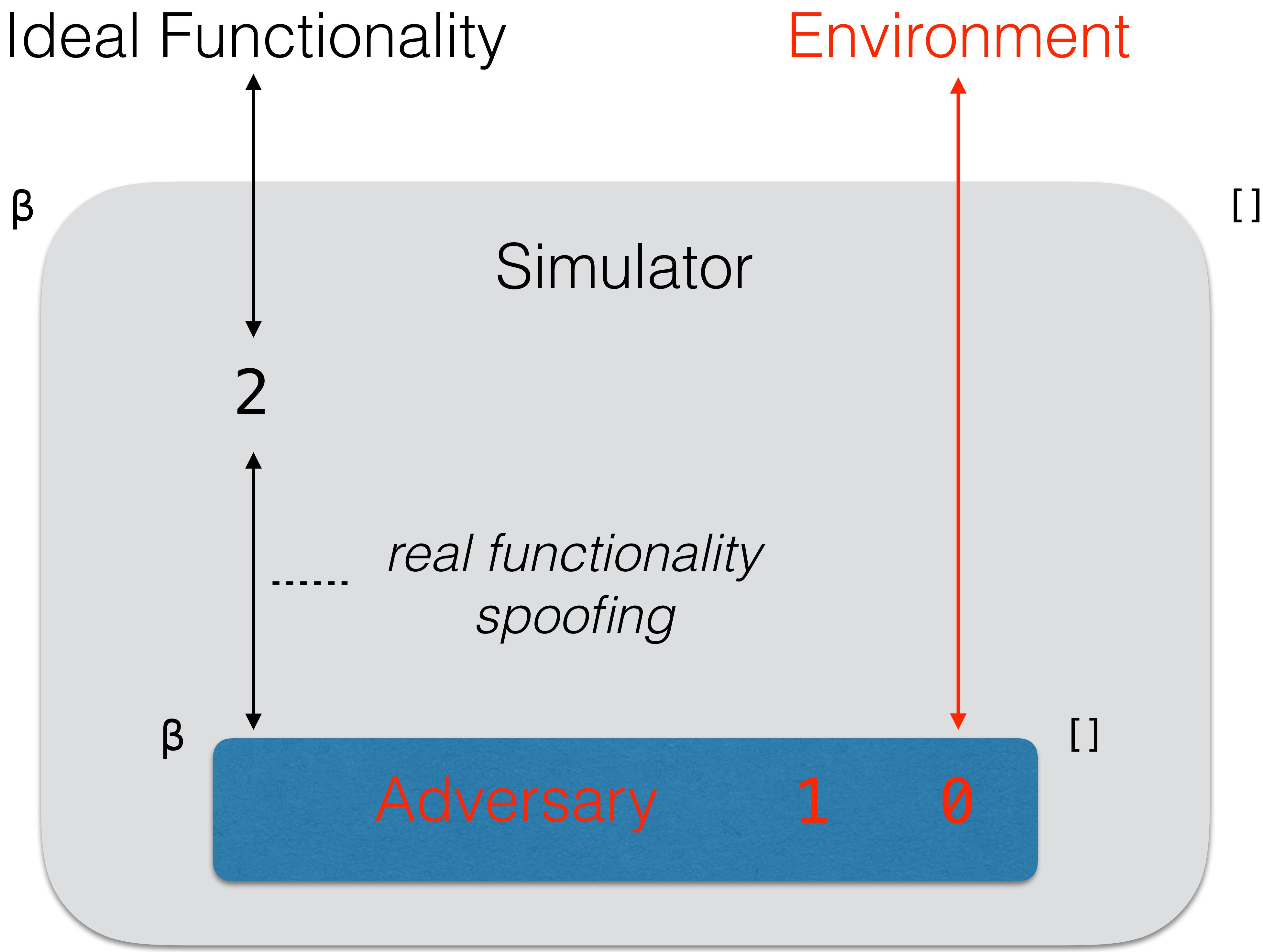


Adv may send *adversarial* messages to **Env**

Simulators


Ideal Functionality
Environment
β
[]
Simulator
2
real functionality
spoofing
β
[]
Adversary
1
0

Simulators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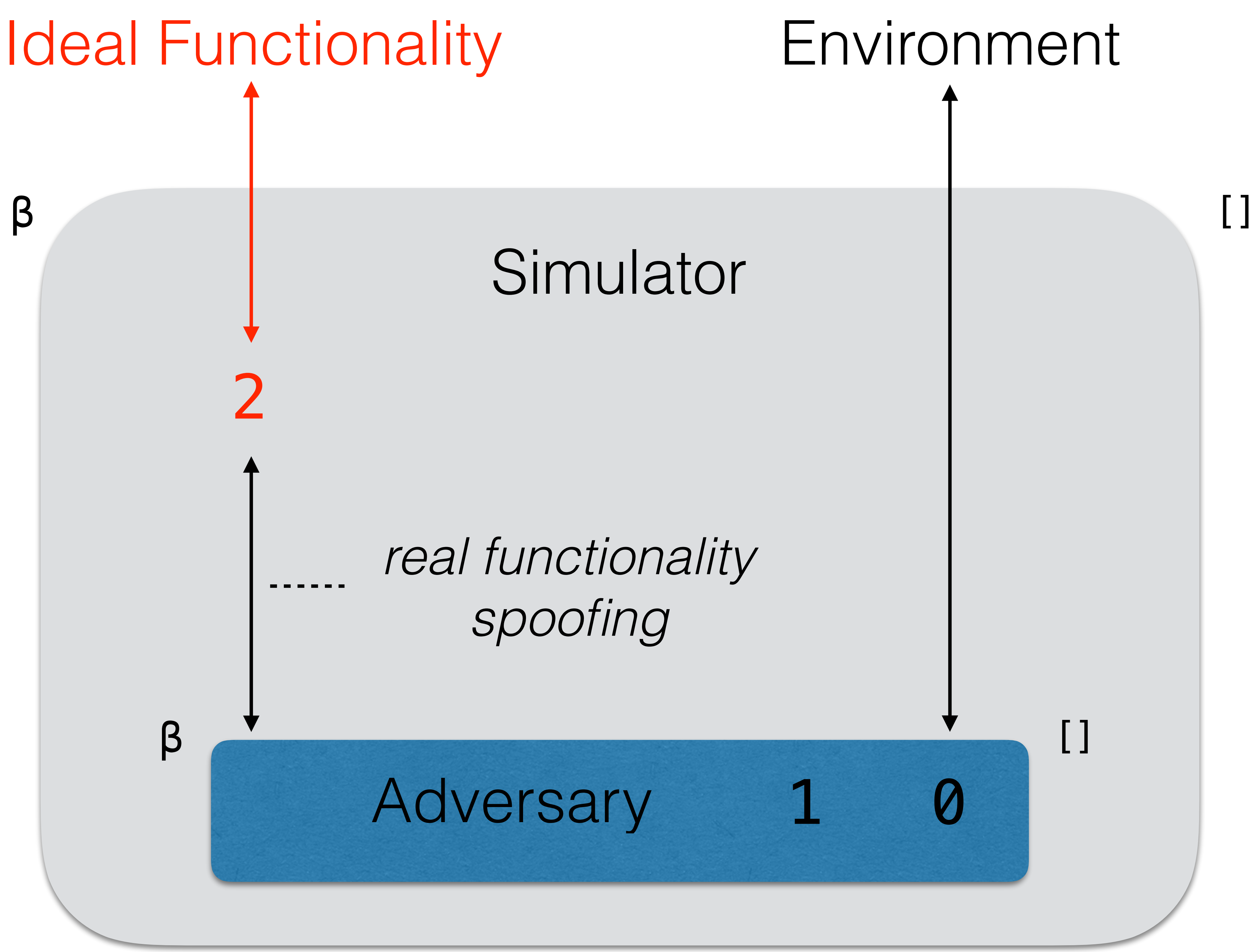
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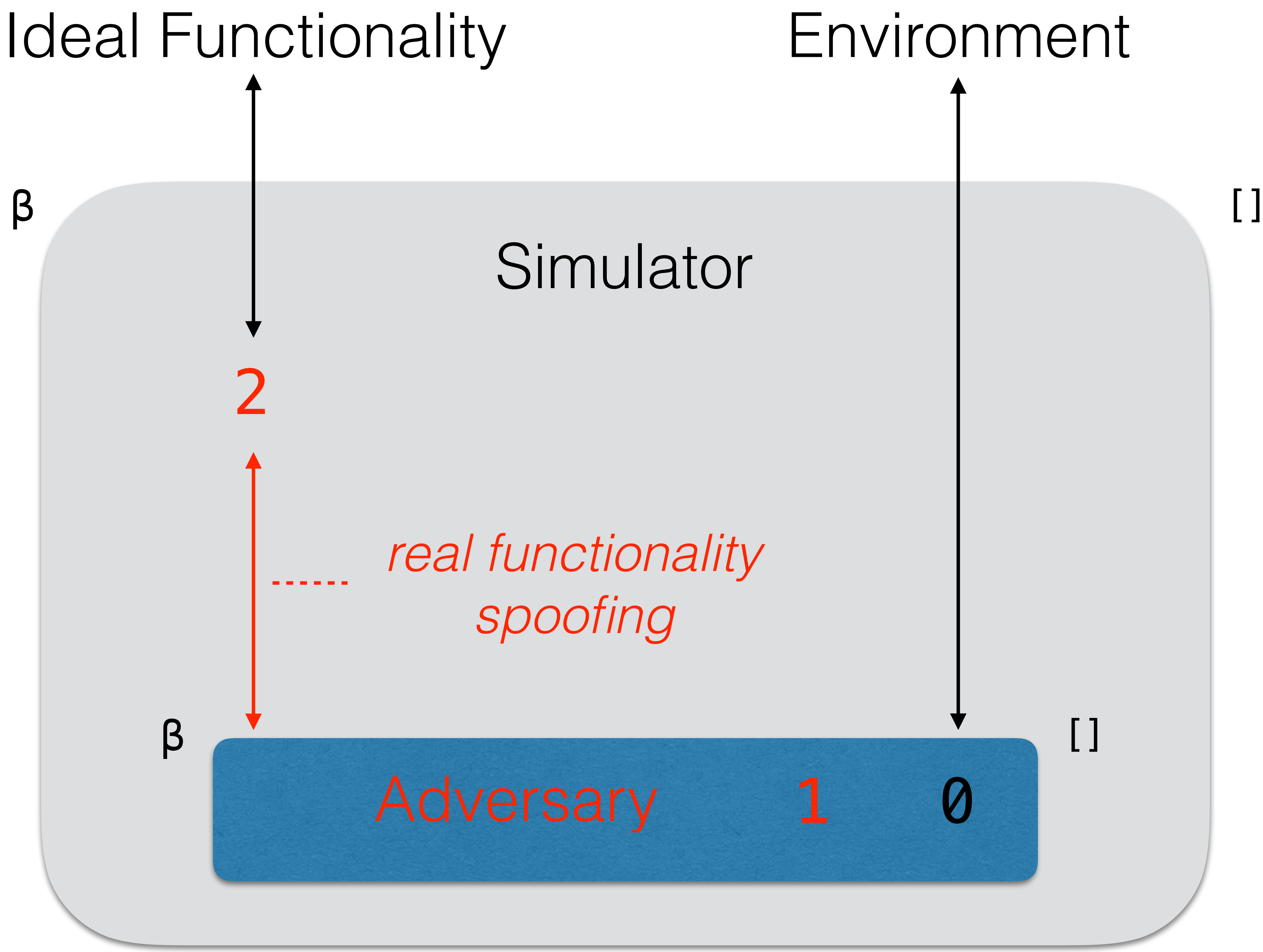
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Secure Message Communication

As a case study, we proved the security of secure message communication in a UC style, via a one-time pad agreed by the parties using Diffie-Hellman key exchange

Group **`key`** of Keys
(**`^^`**, **`id`**, **`inv`**, **`g`**)

k ^ q : key

Commutative Semi-group
`exp` of Exponents
(∗)

inj

proj (partial)

Type **`text`** of plain texts

`g` ^ q determines all keys, uniquely

`(` k **`^`**q**`)^`**r **`=`** k **`^(`** q∗r **`)`**

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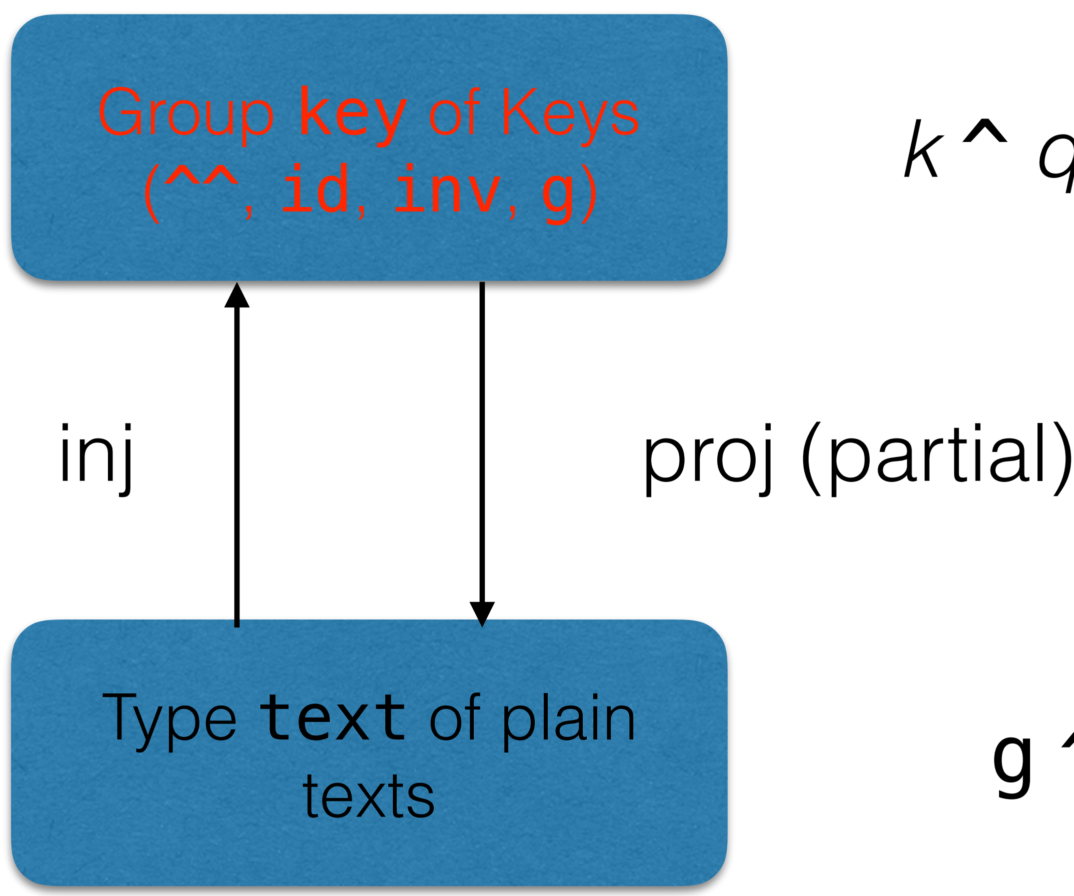

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(k ^ q)^ r = k ^(q ∗ r)

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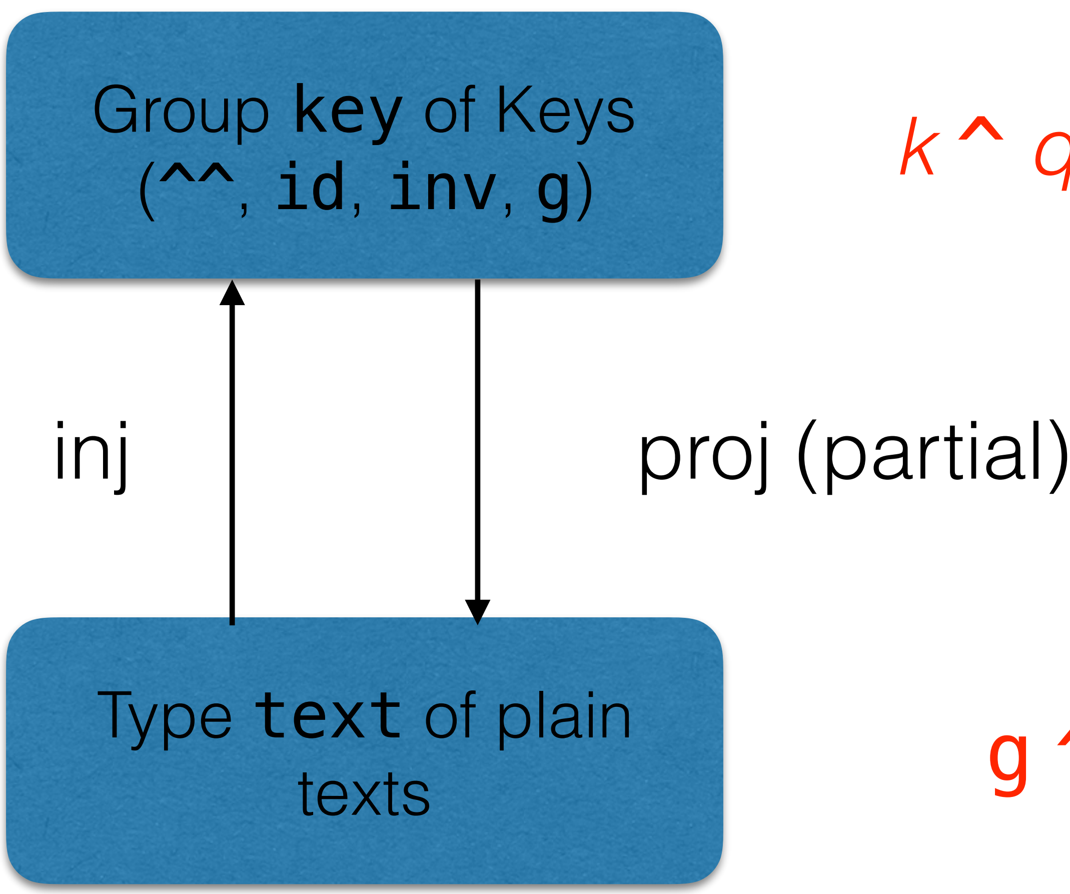

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$(k \,\texttt{\^{}}\, q) \,\texttt{\^{}}\, r \;=\; k \,\texttt{\^{}}\, (q * r)$

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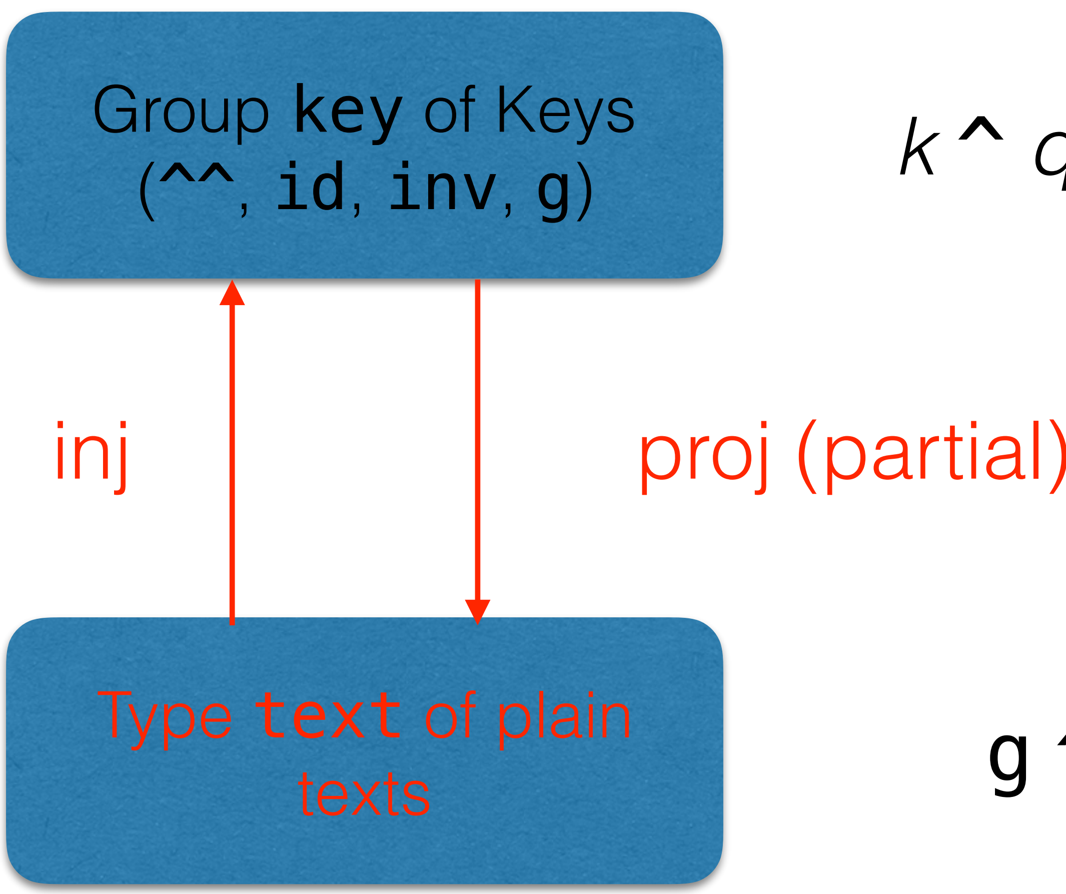

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`(`k **`^`**q**`)`**`^`r **`=`** k **`^(`** q∗r**`)`**

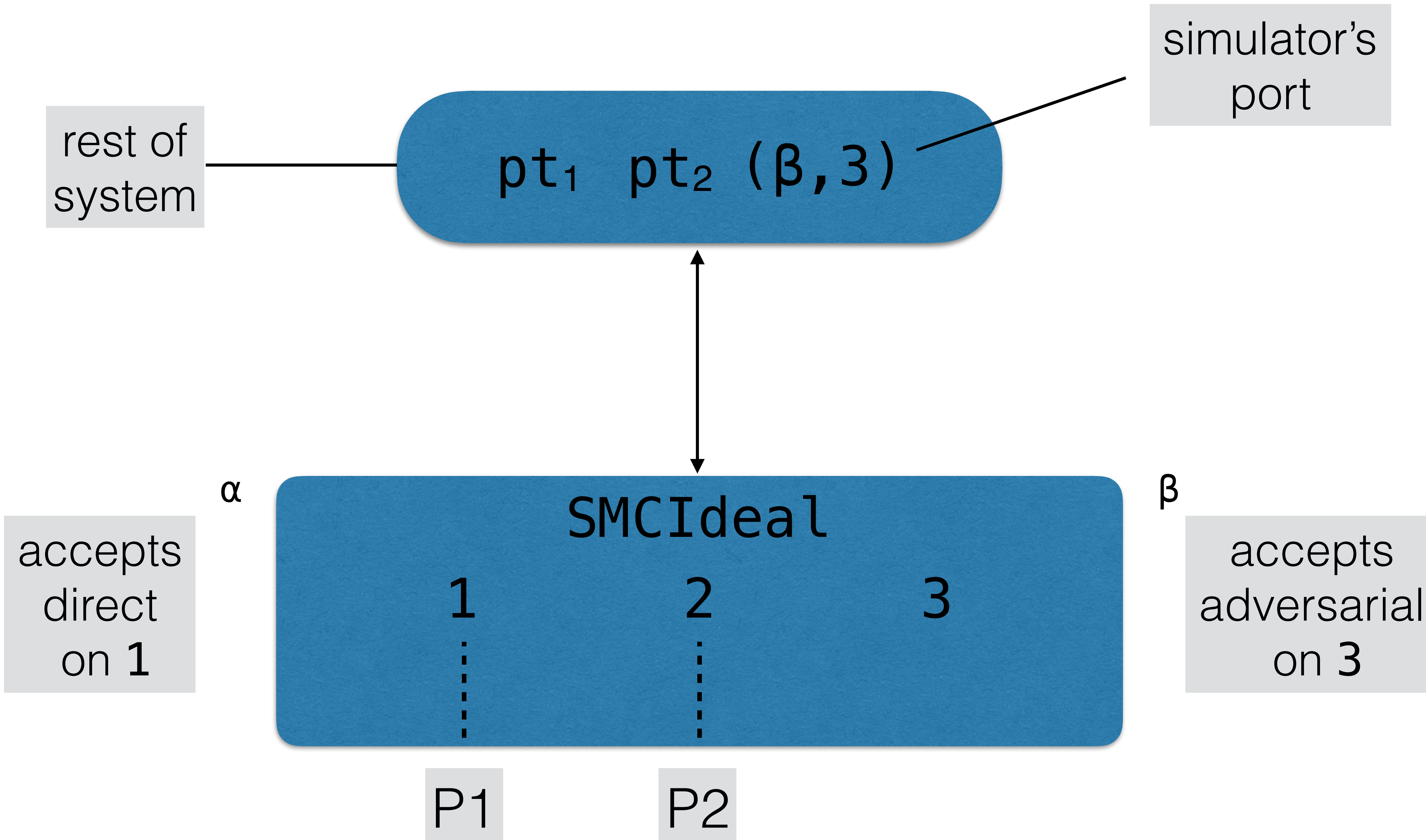
Secure Message Communication

- Two protocol parties: 1 and 2
  - P1 wants to securely transmit plain text *t* to P2
- P1 and P2 use Diffie-Hellman key exchange to agree on a key, *k* — *(see next slide)*
- P1 transmits *e* `= inj` *t* `^^` *k* to P2 — adversary observes but can't corrupt
- P2 gets decryption of *e* as `proj (`*e* `^^ inv` *k*`)`

Diffie-Hellman Key Exchange

- P1 and P2 both have their own randomly generated secrets q_1, q_2 : exp
- P1 sends $\texttt{g\^{}}q_1$ to P2, which sends $\texttt{g\^{}}q_2$ to P1 — adversary observes these transmissions
- P1 then computes $(g\text{\^{}}q_2)\text{\^{}}q_1 = g\text{\^{}}(q_2 * q_1) = g\text{\^{}}(q_1 * q_2)$ as the shared key, k
- P2 then computes $(g\text{\^{}}q_1)\text{\^{}}q_2 = g\text{\^{}}(q_1 * q_2)$ as the shared key, k

SMC Ideal Functionality



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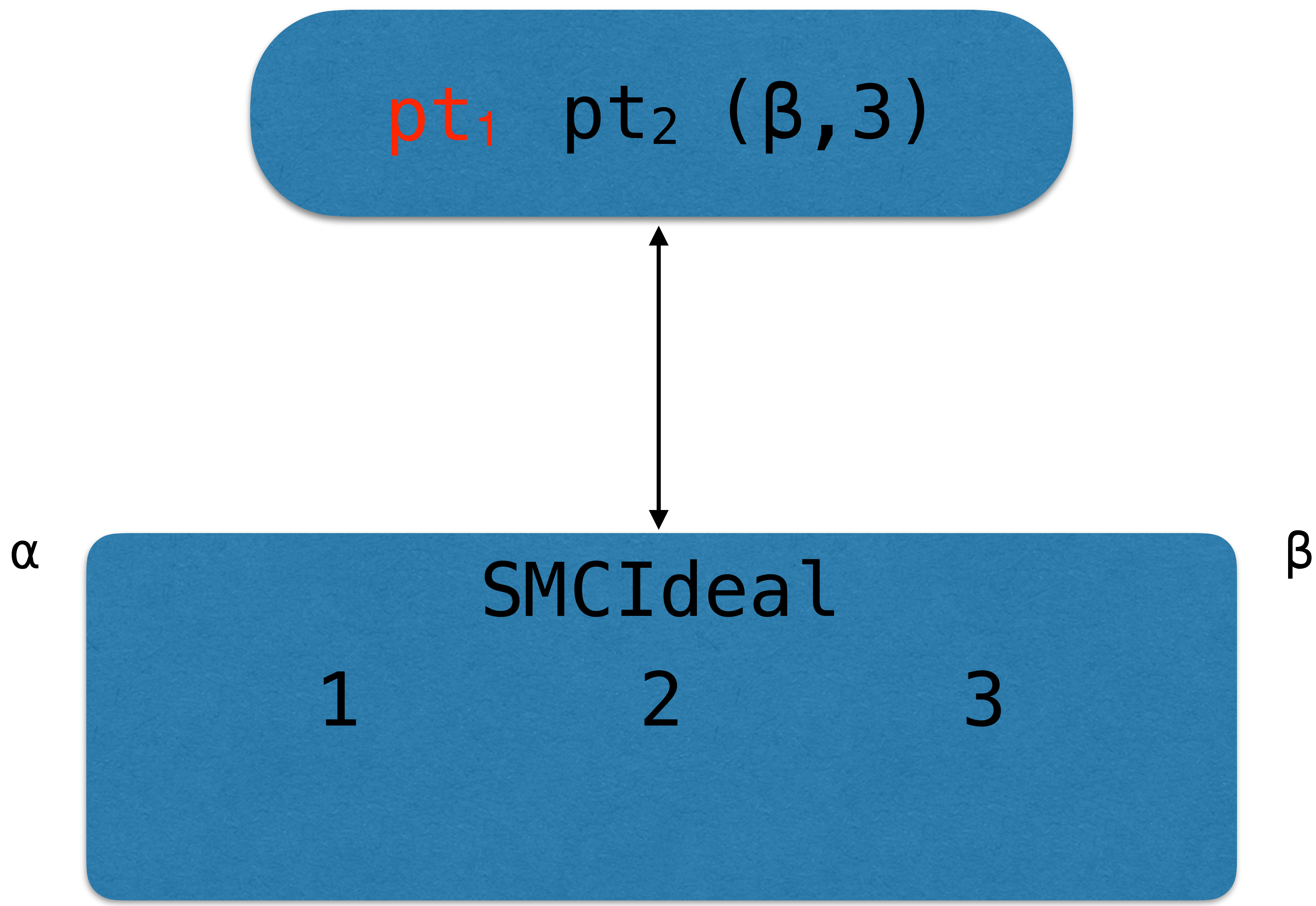

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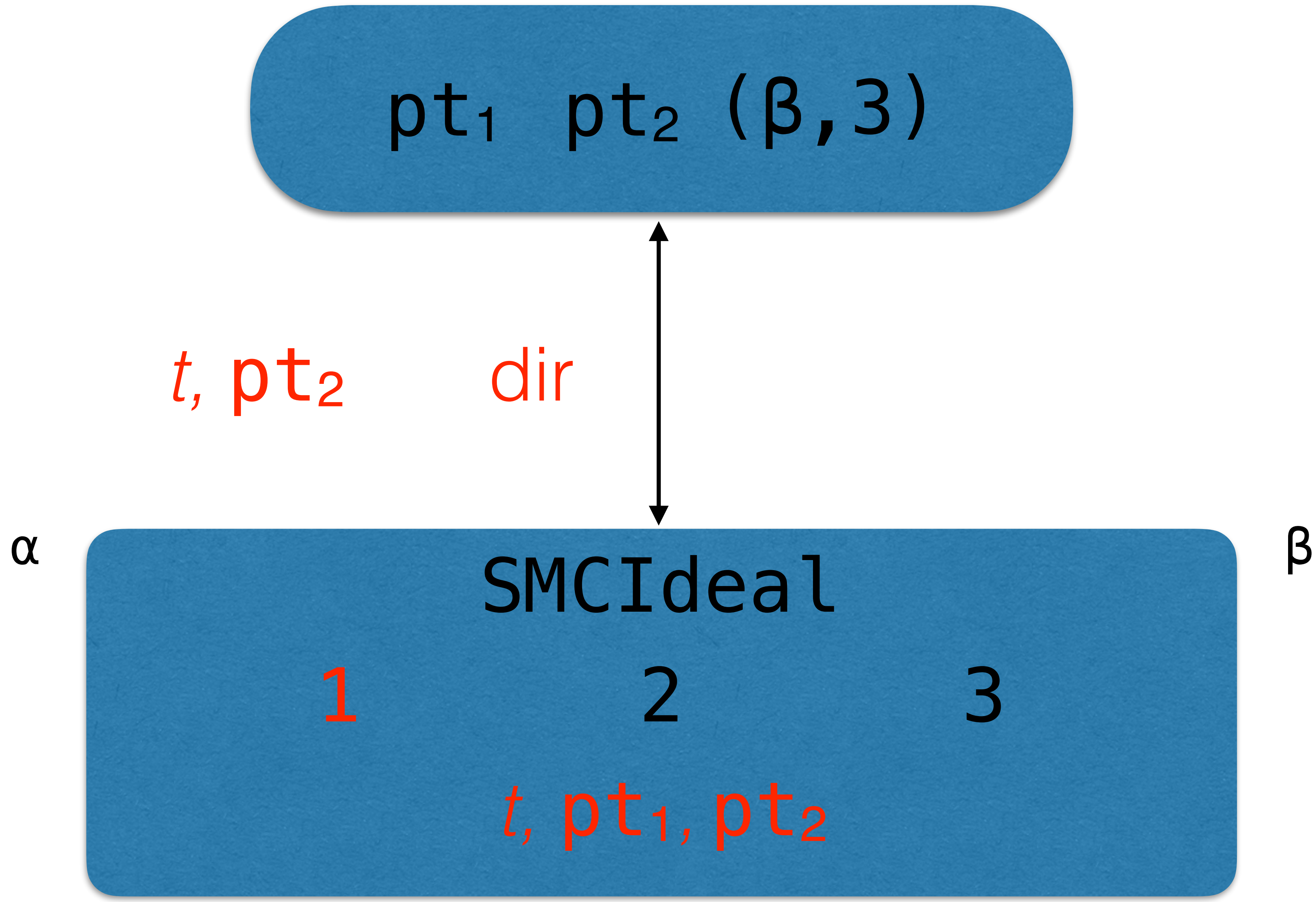

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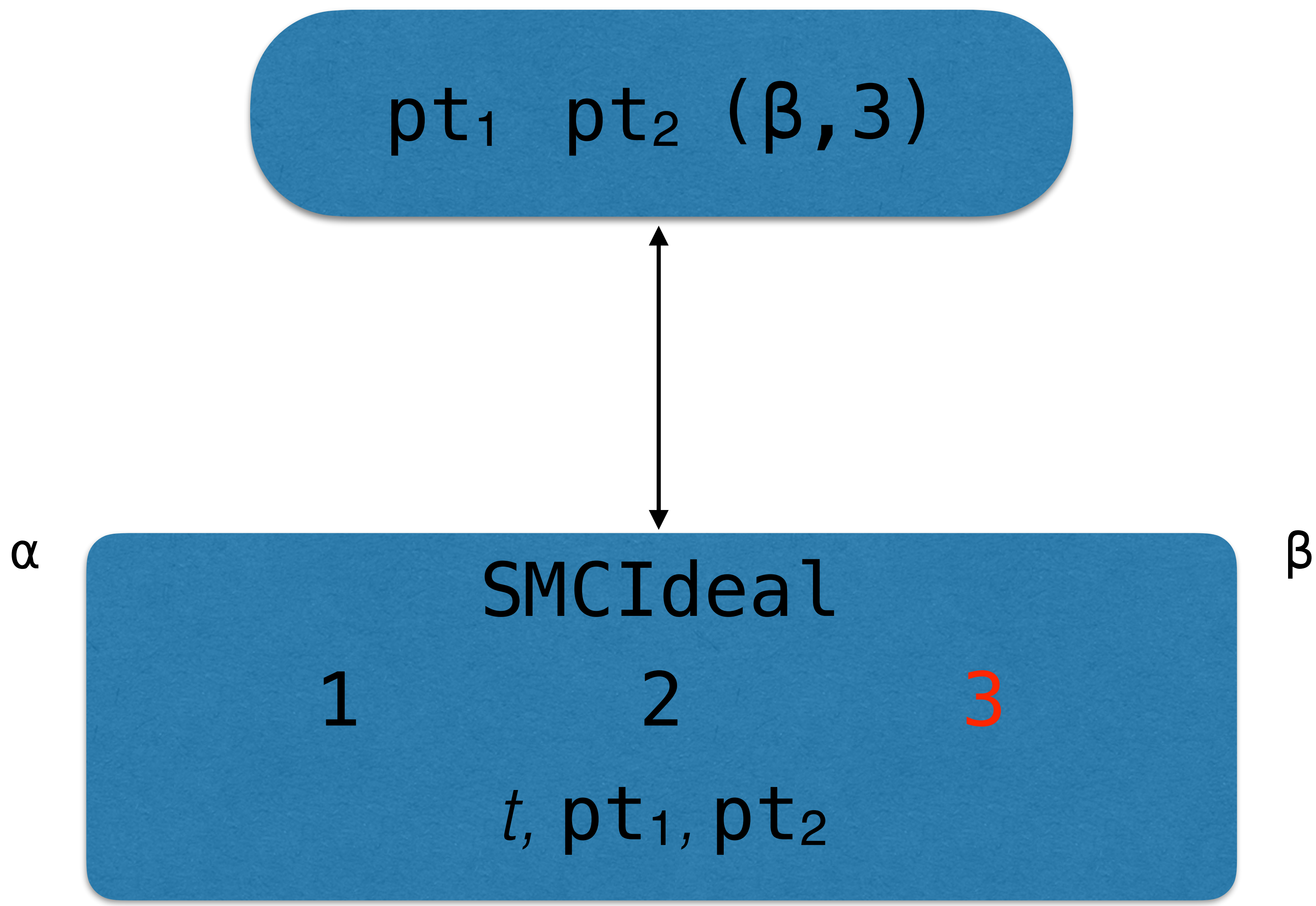
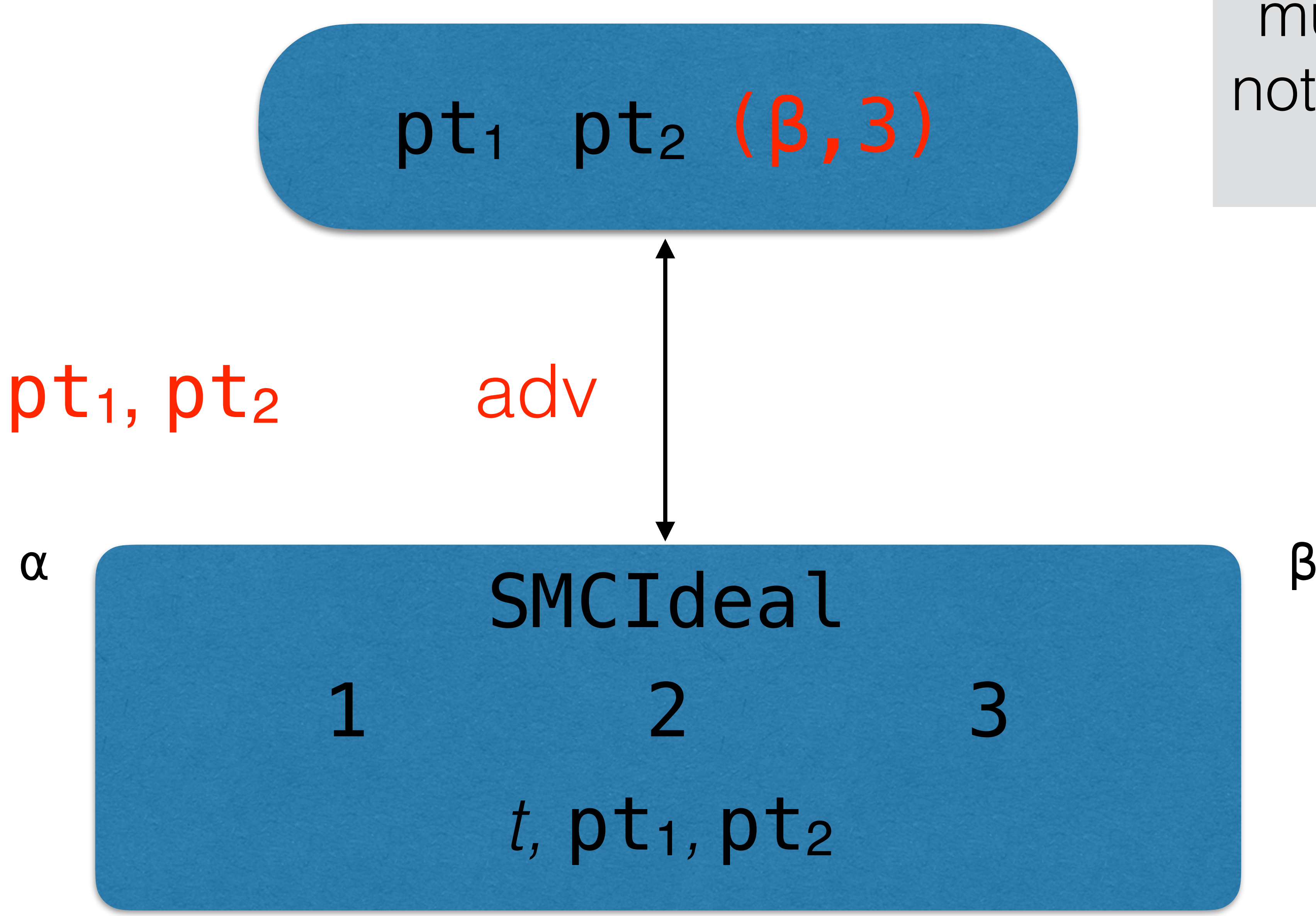


SMC Ideal Functionality

simulator must work not knowing *t*



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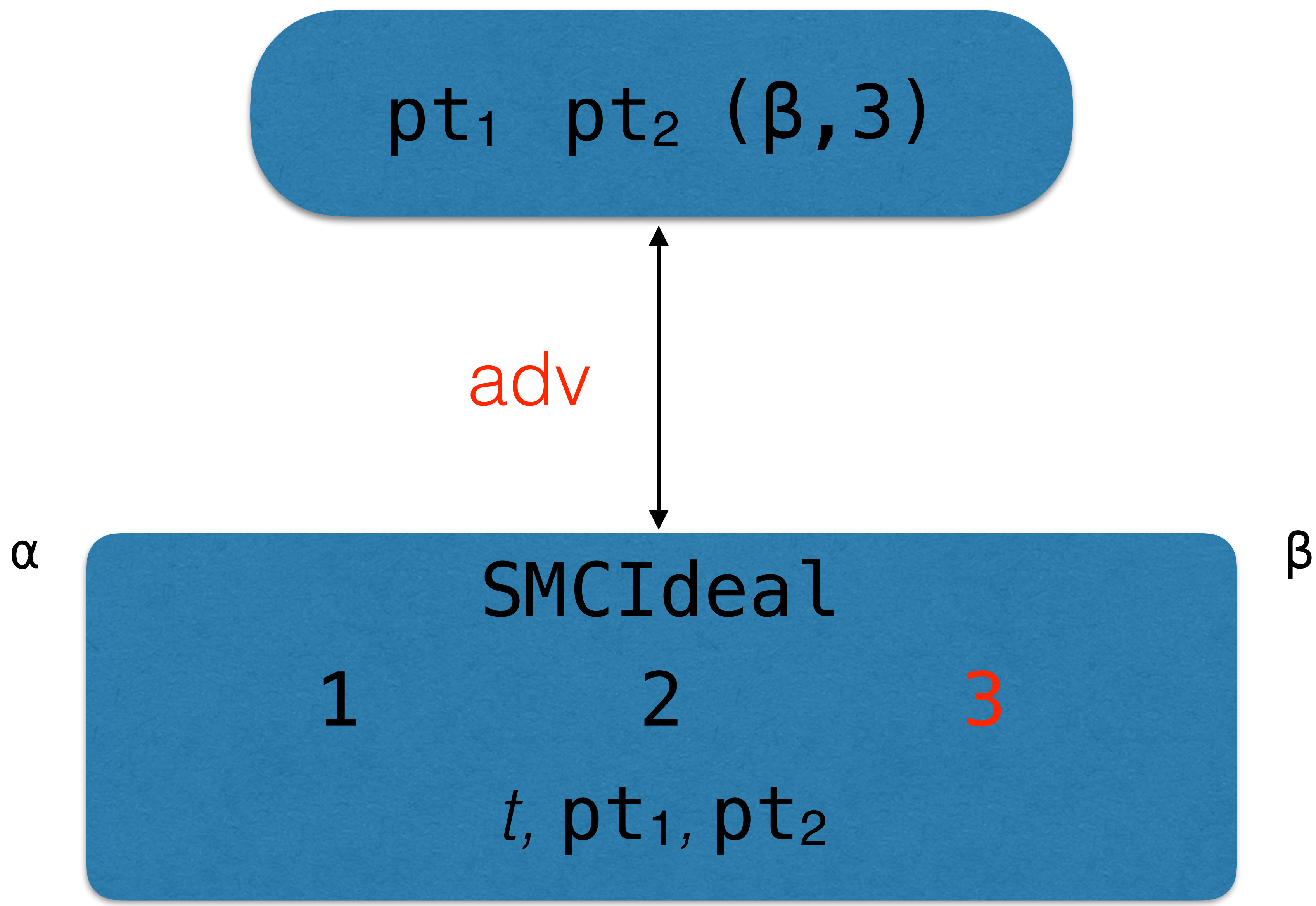

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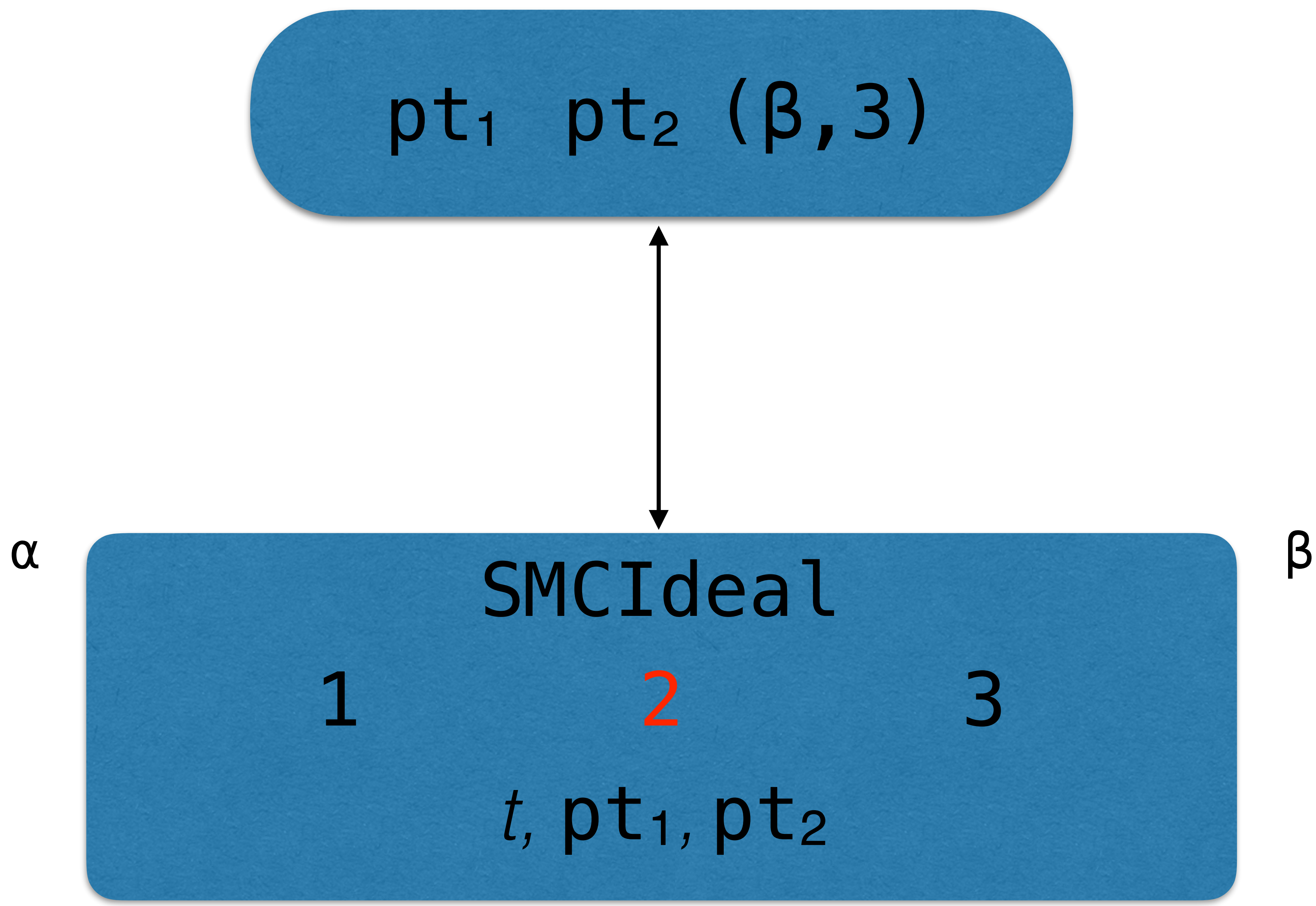

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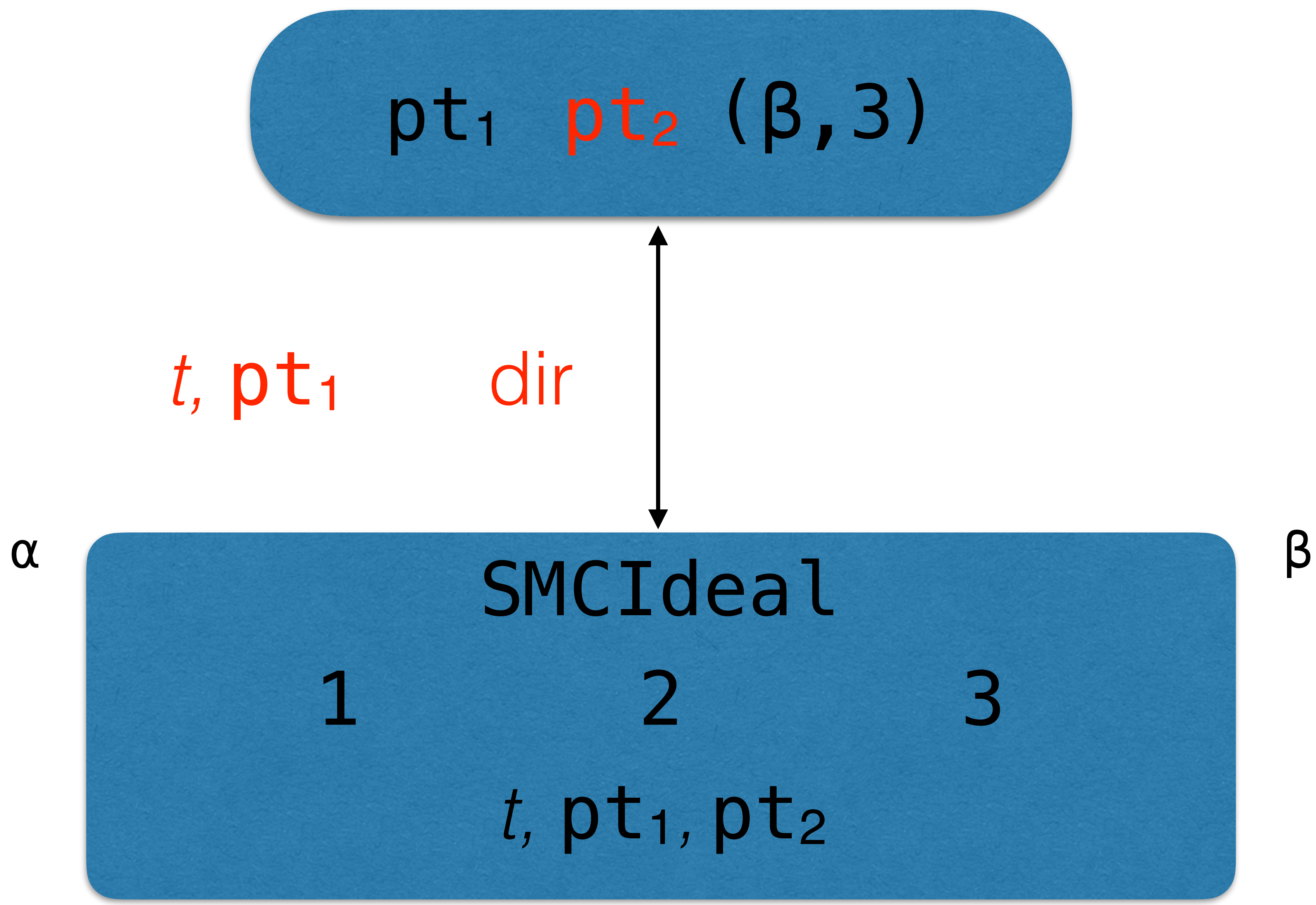
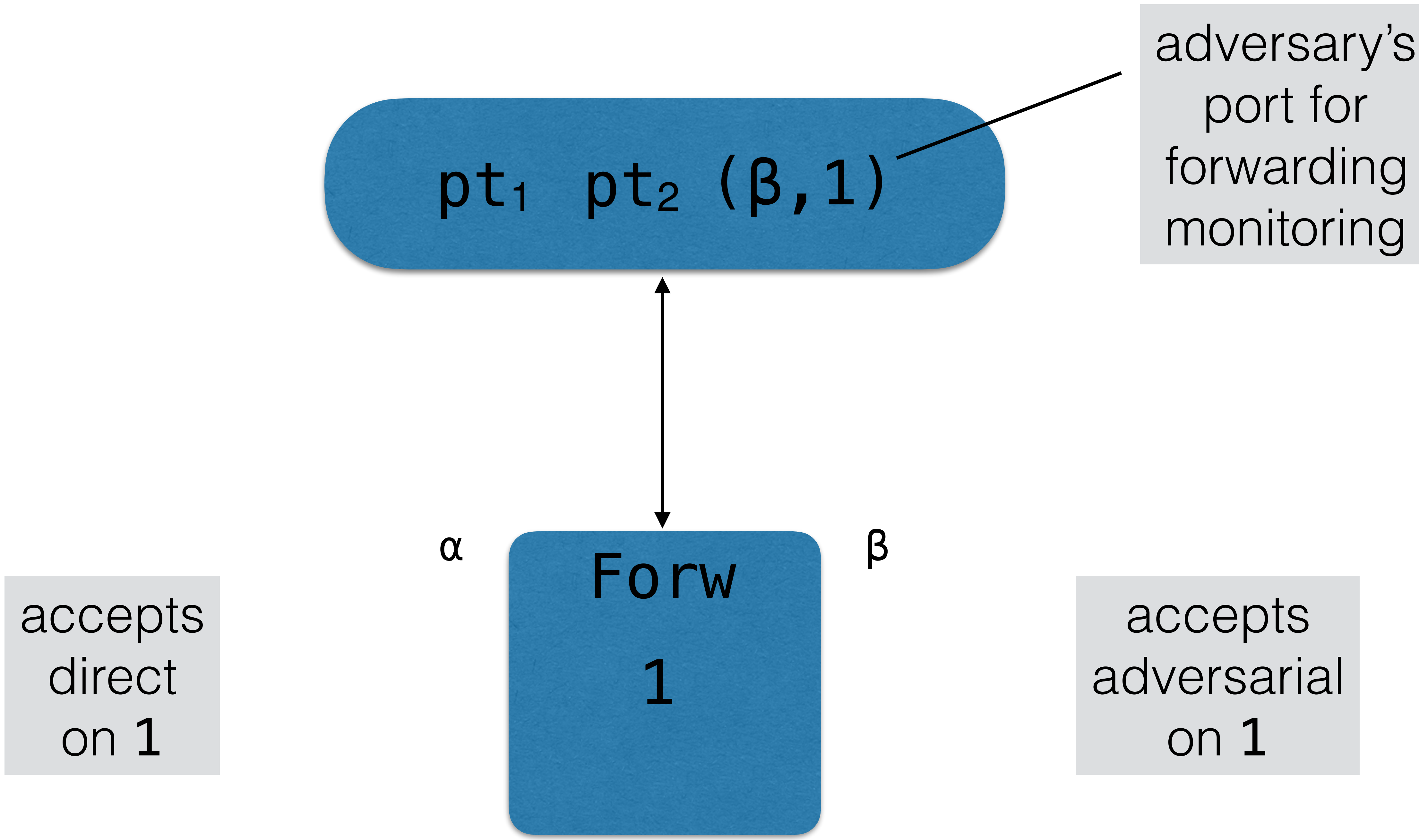


Ideal Forwarding Functionality



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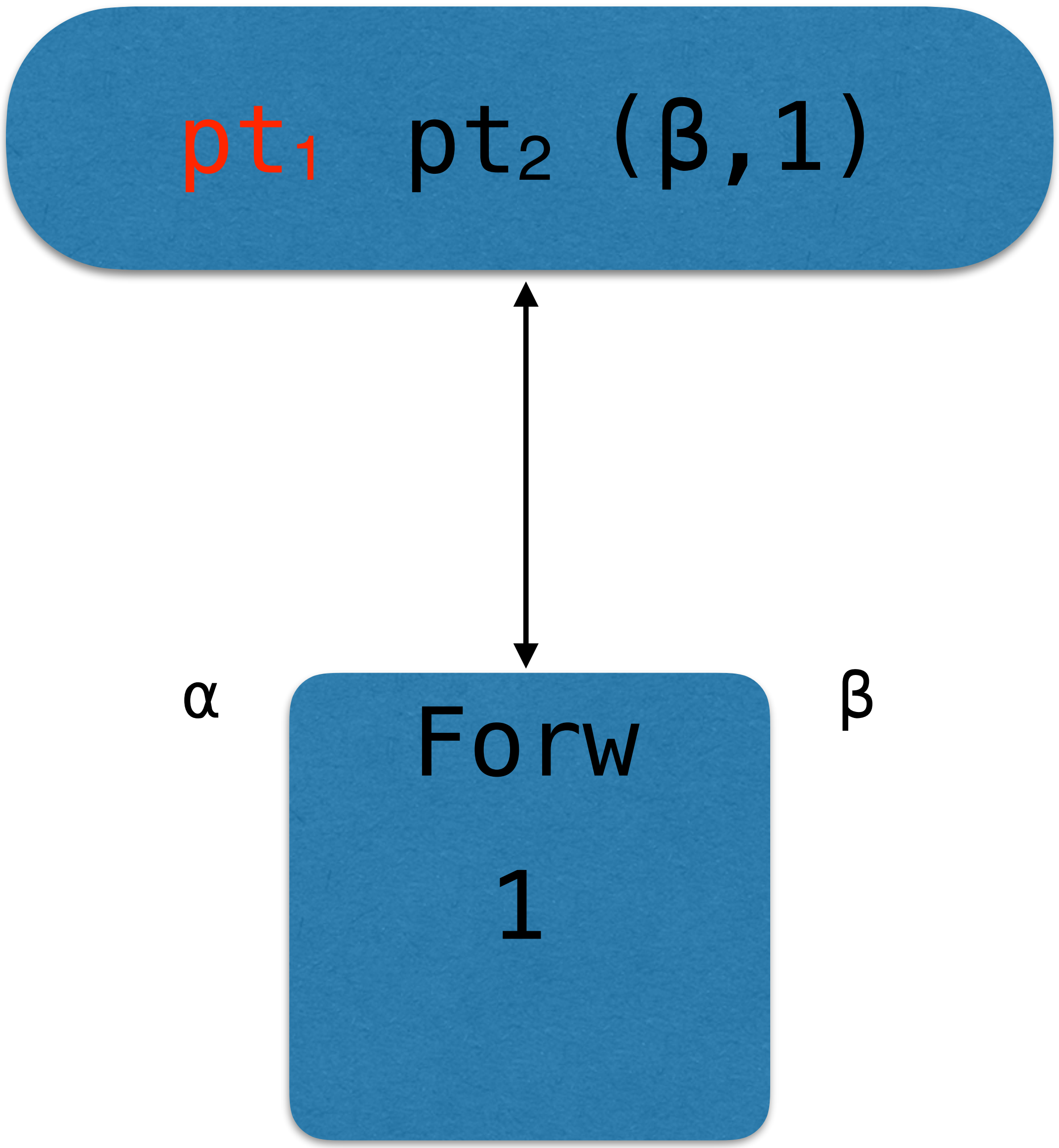

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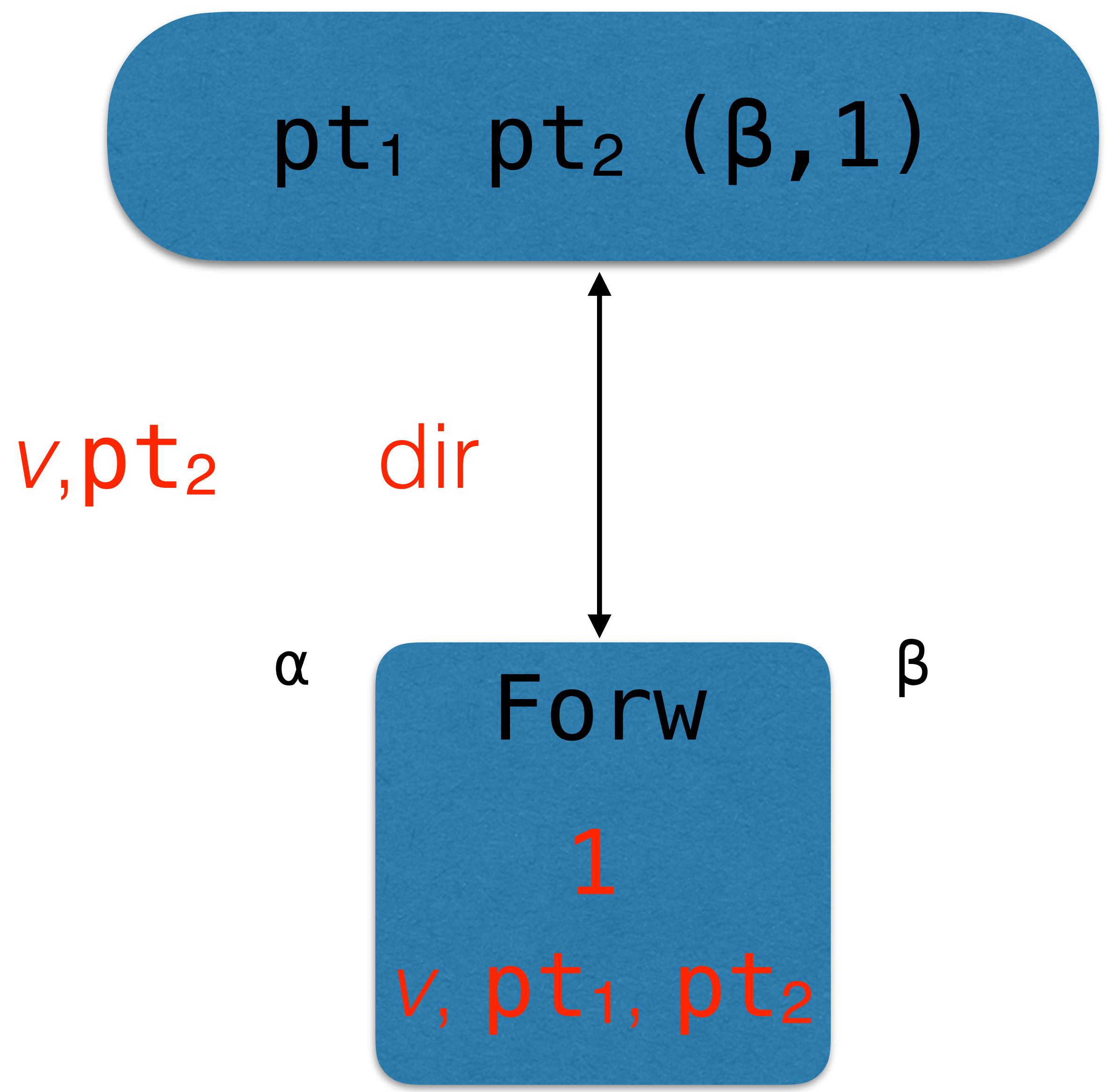

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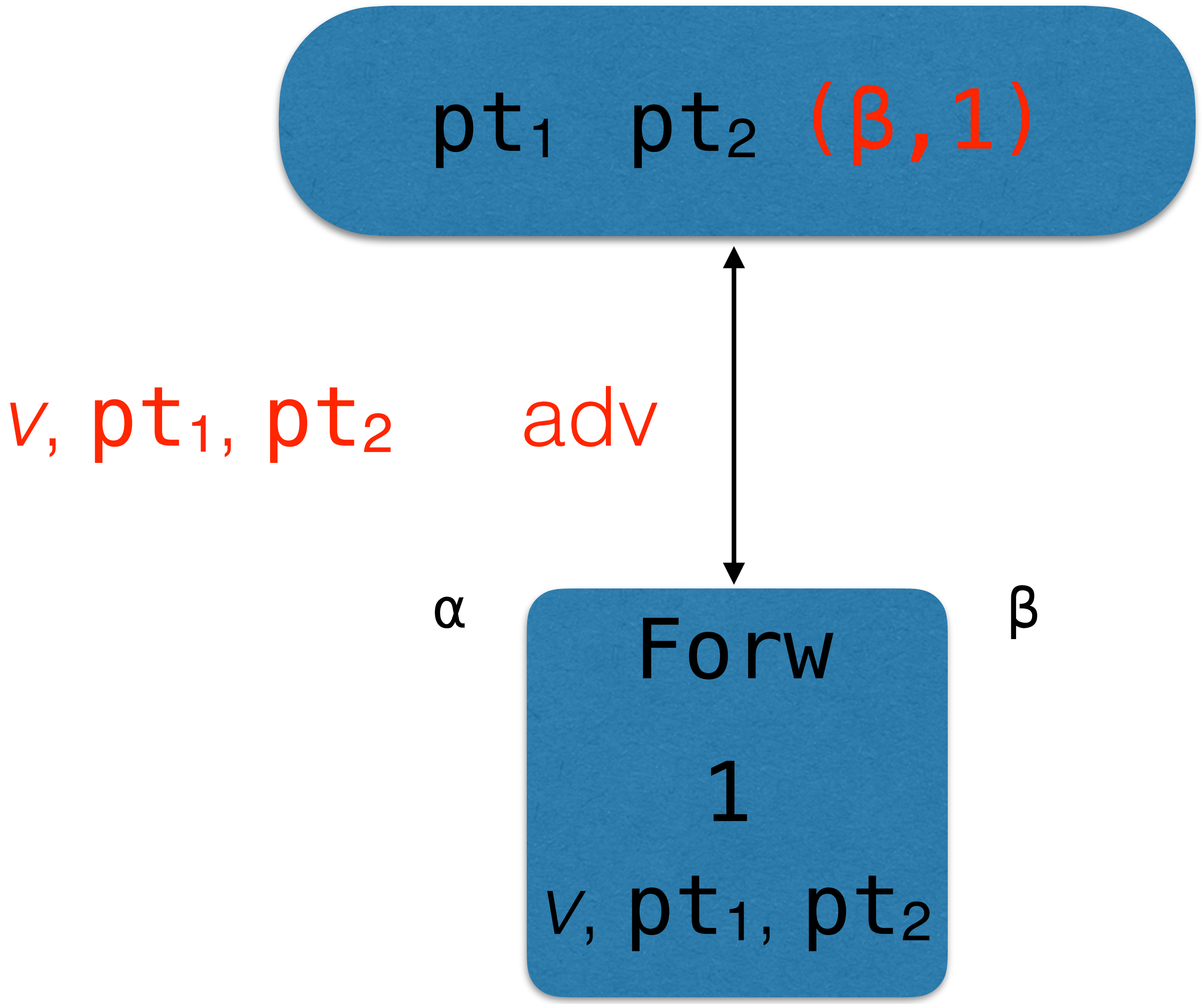

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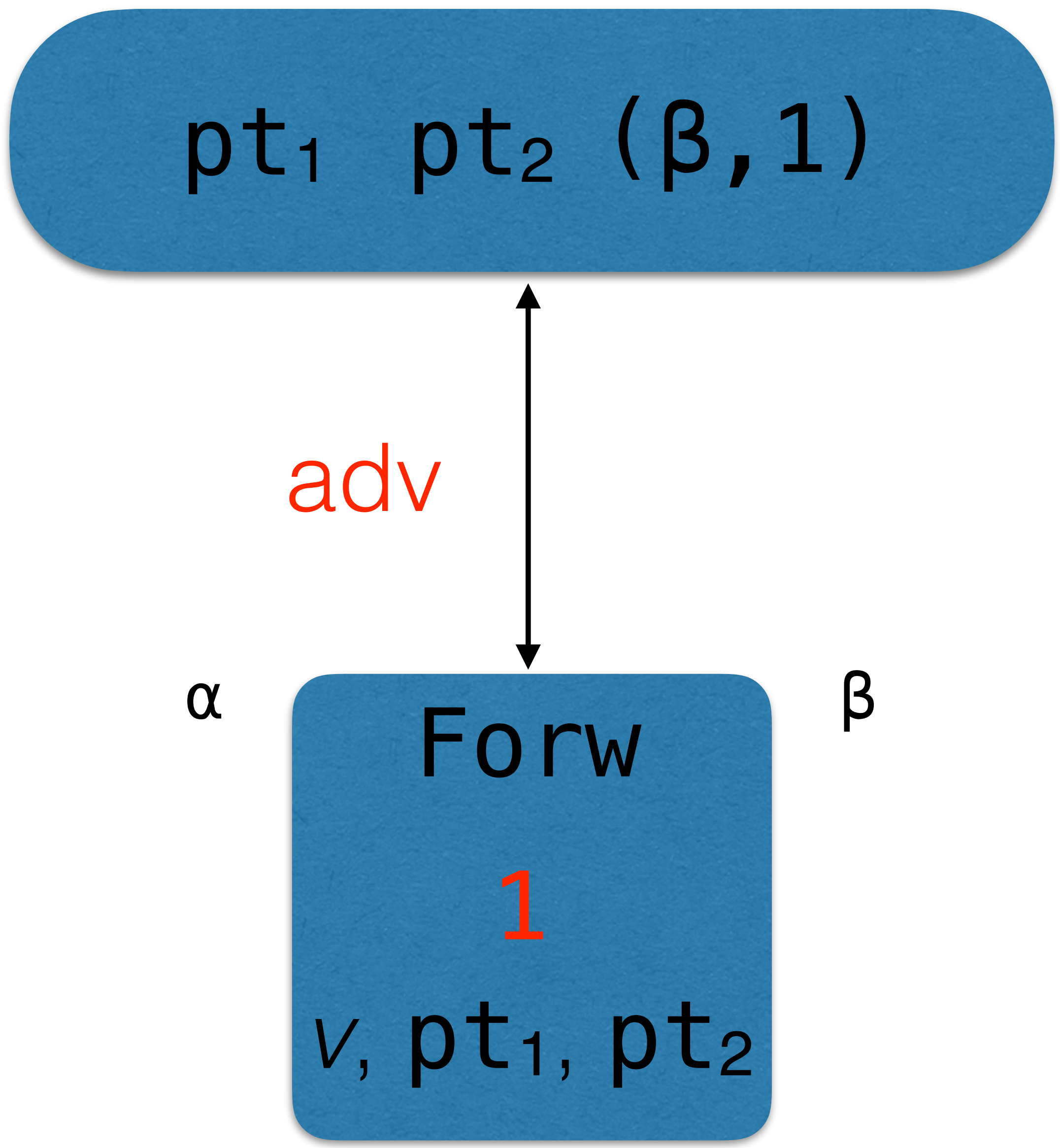

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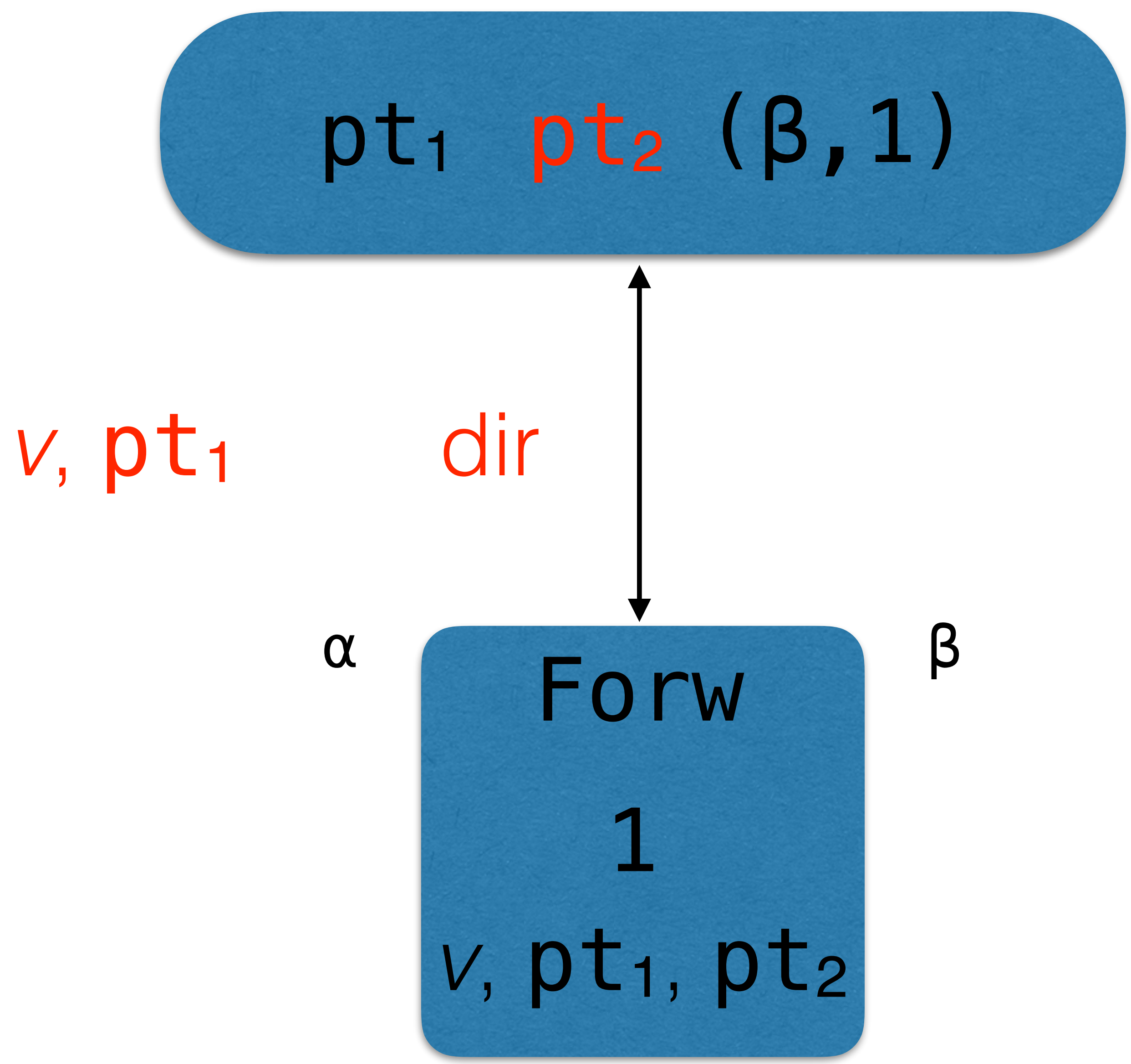
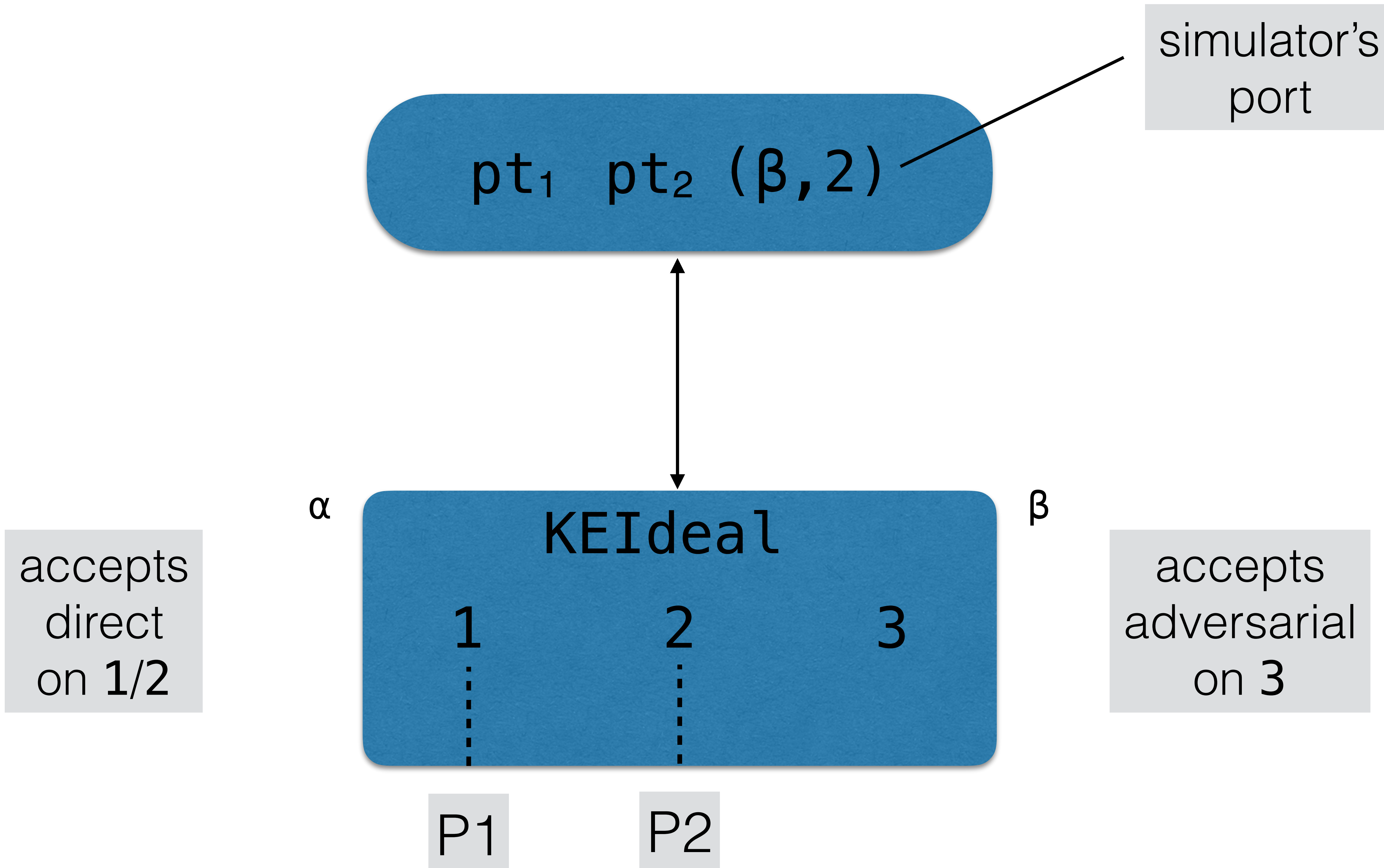


Key Exchange Ideal Functionality



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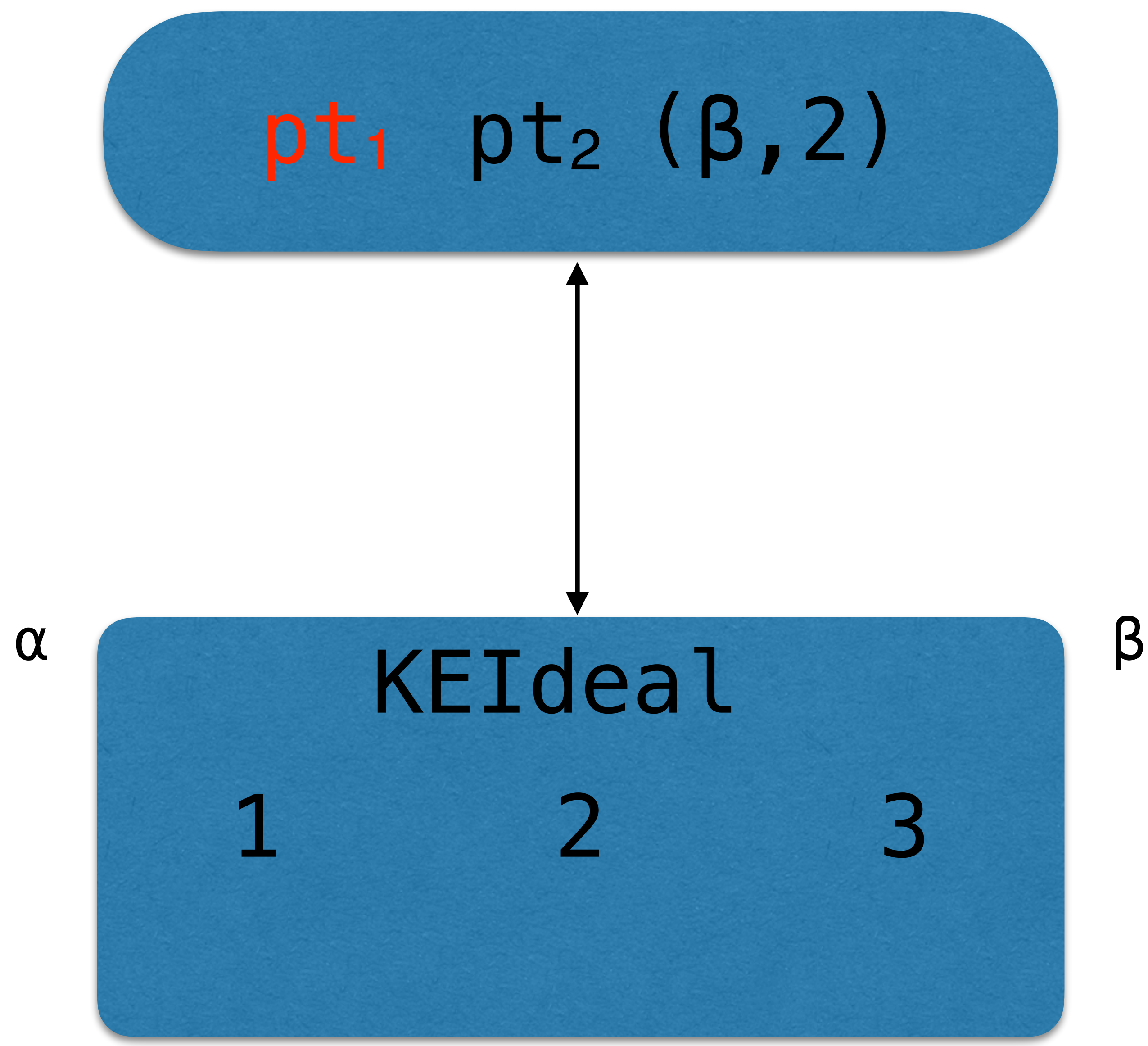

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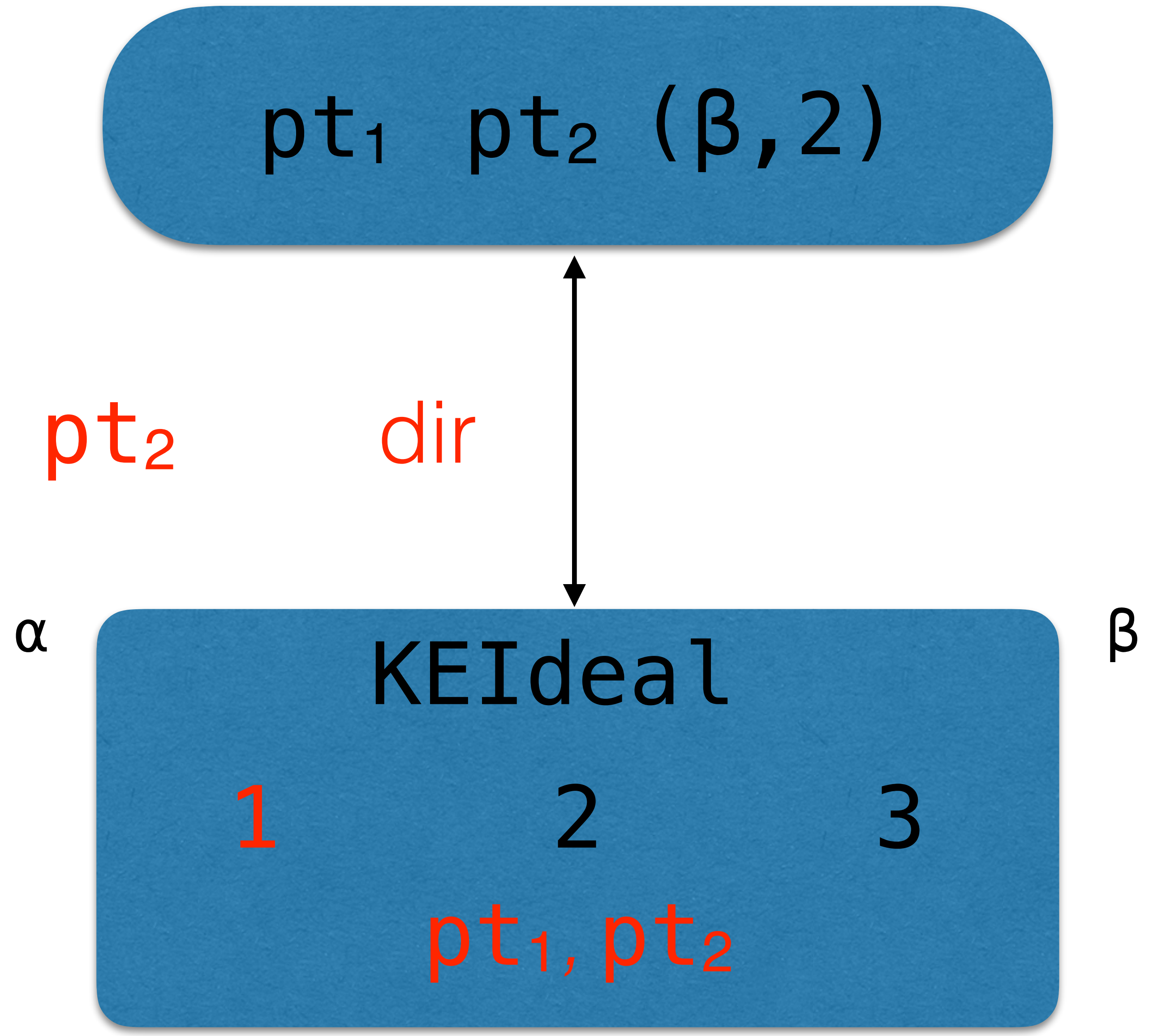

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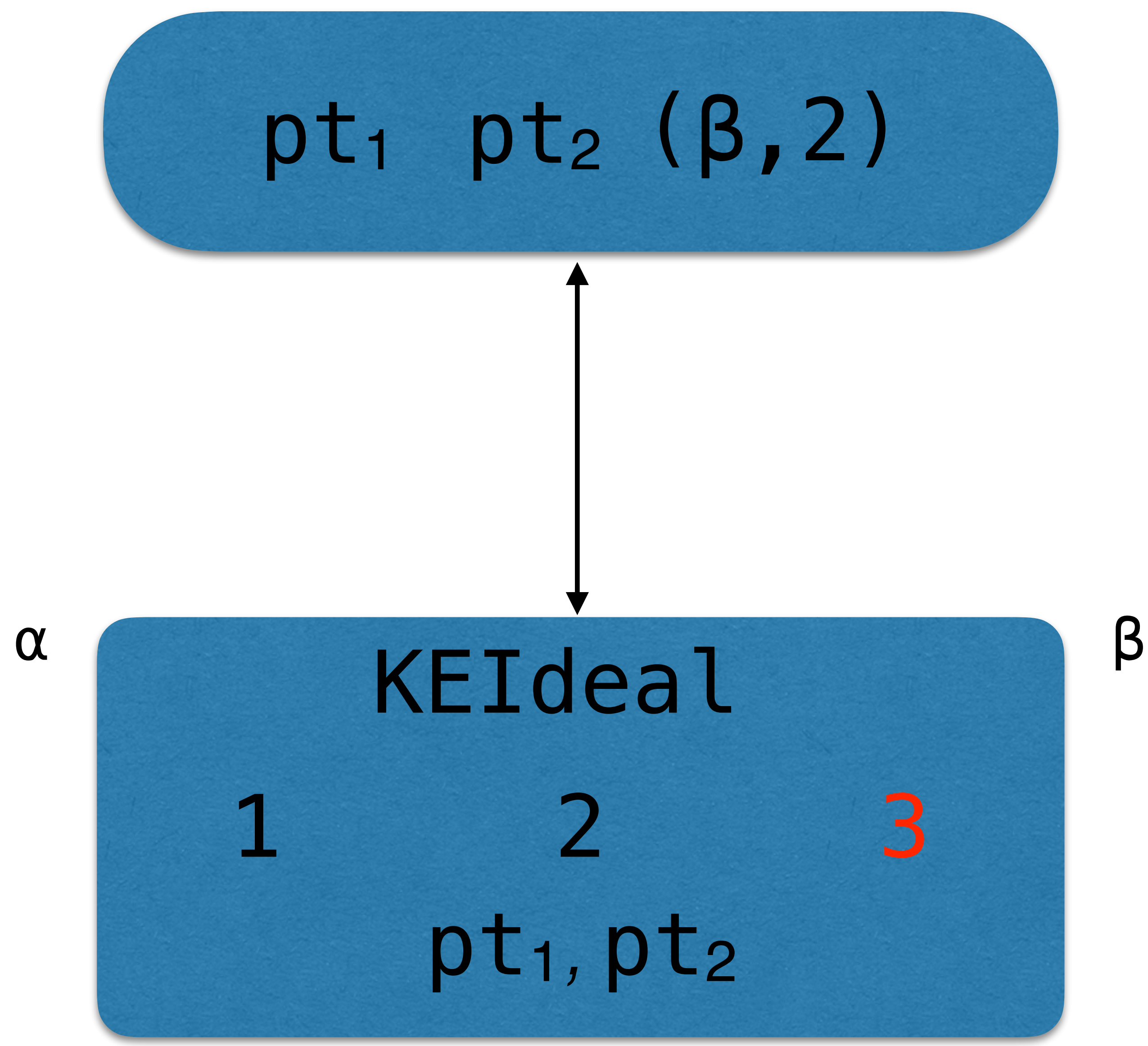

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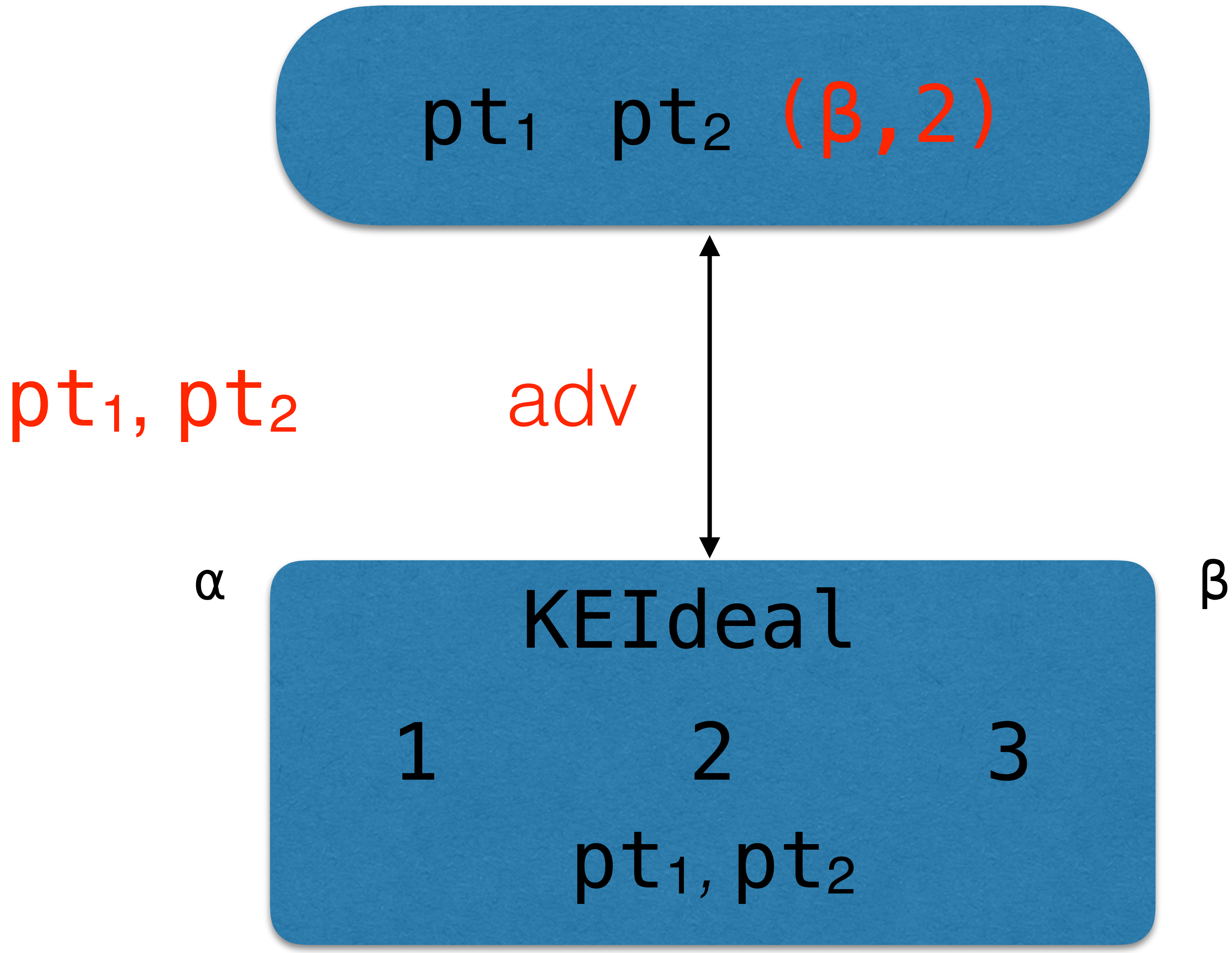

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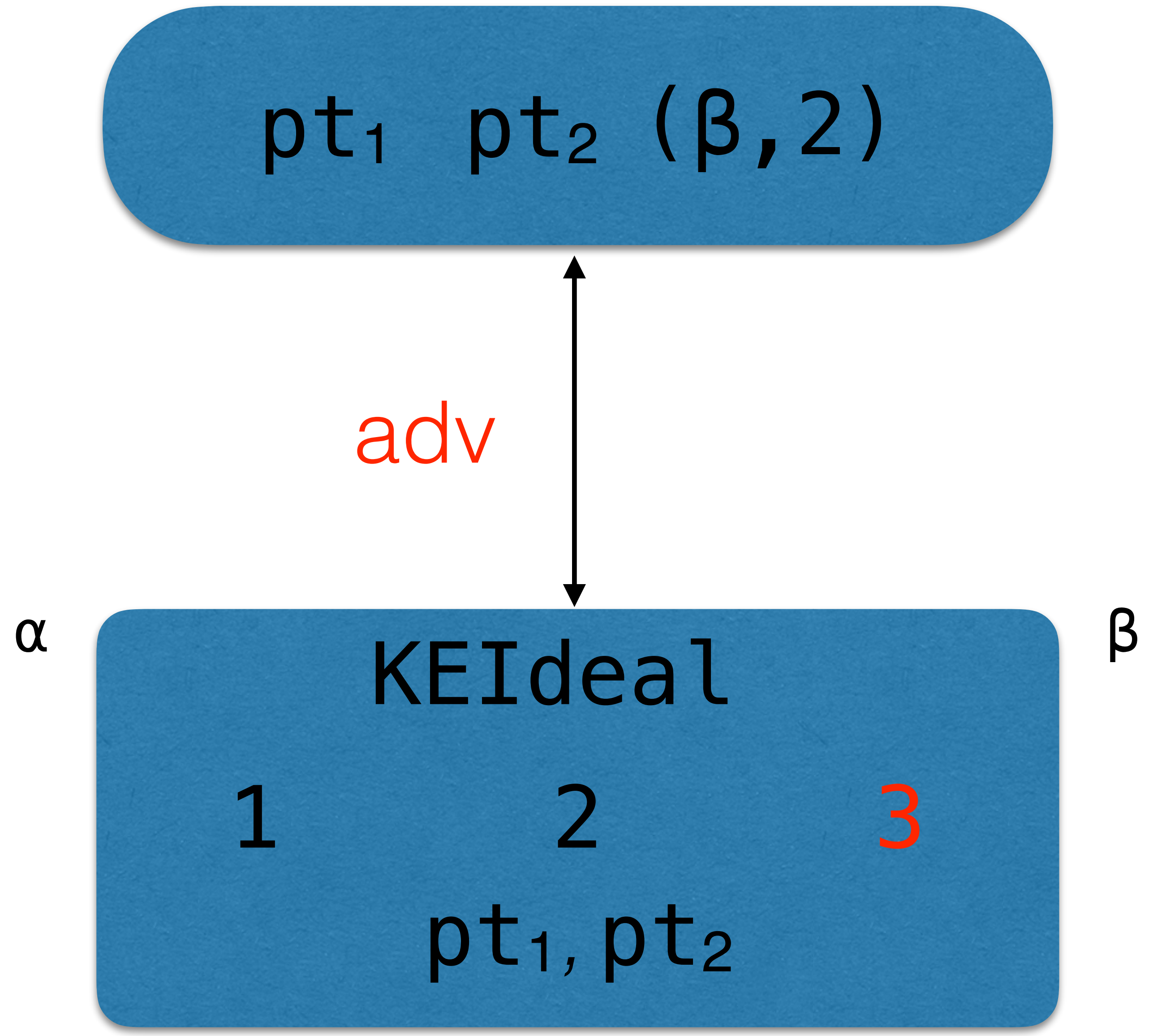

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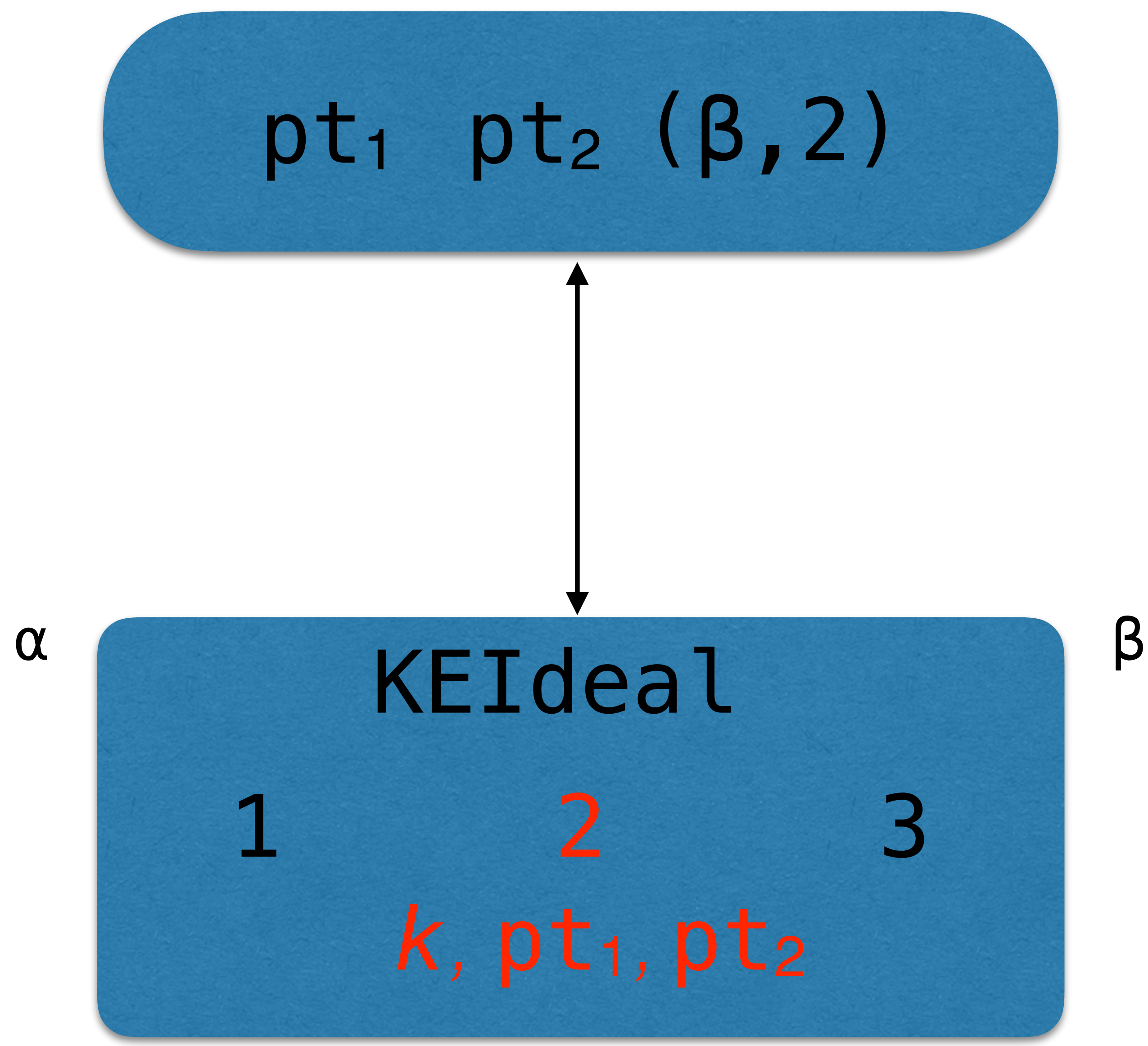

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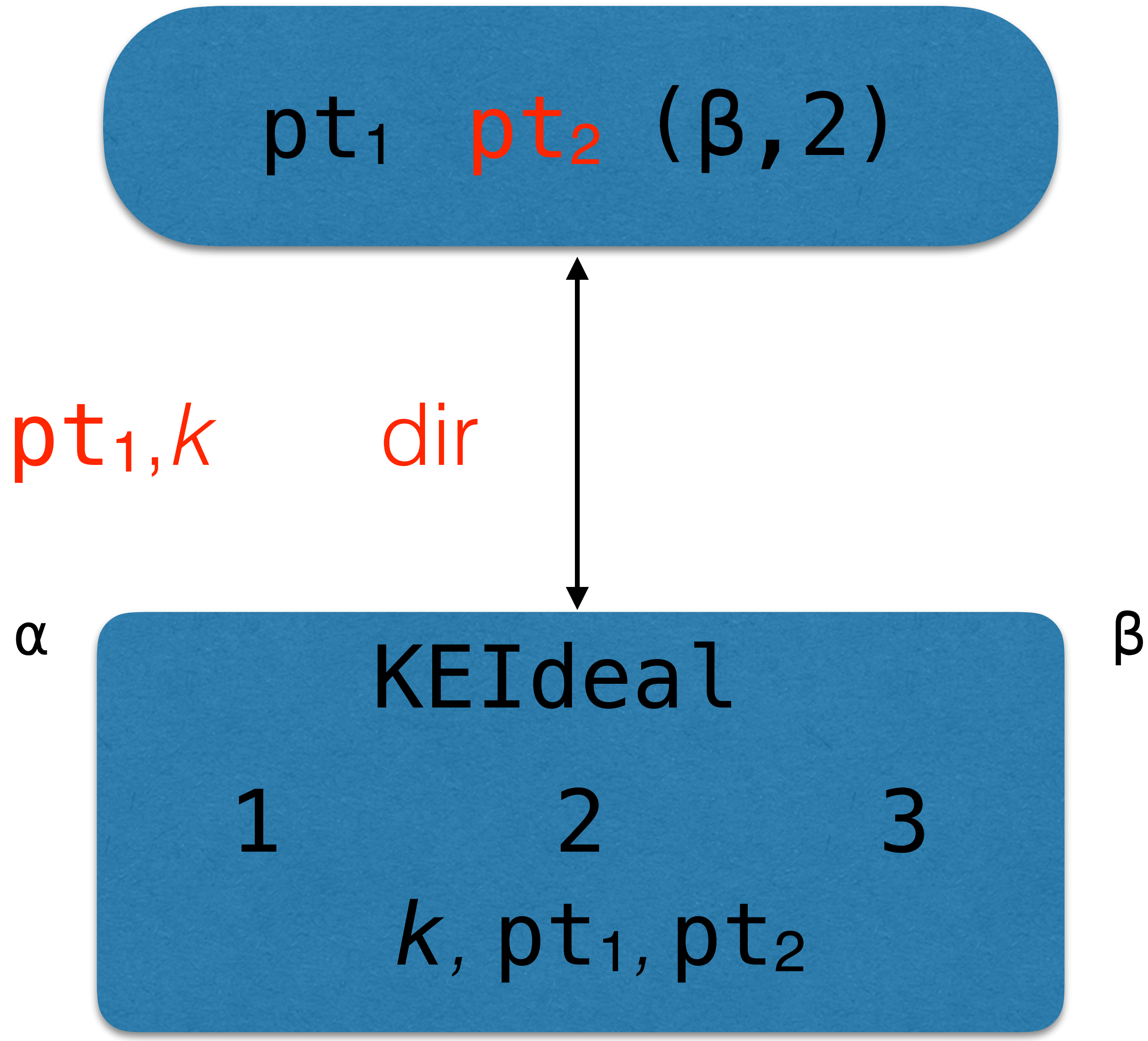

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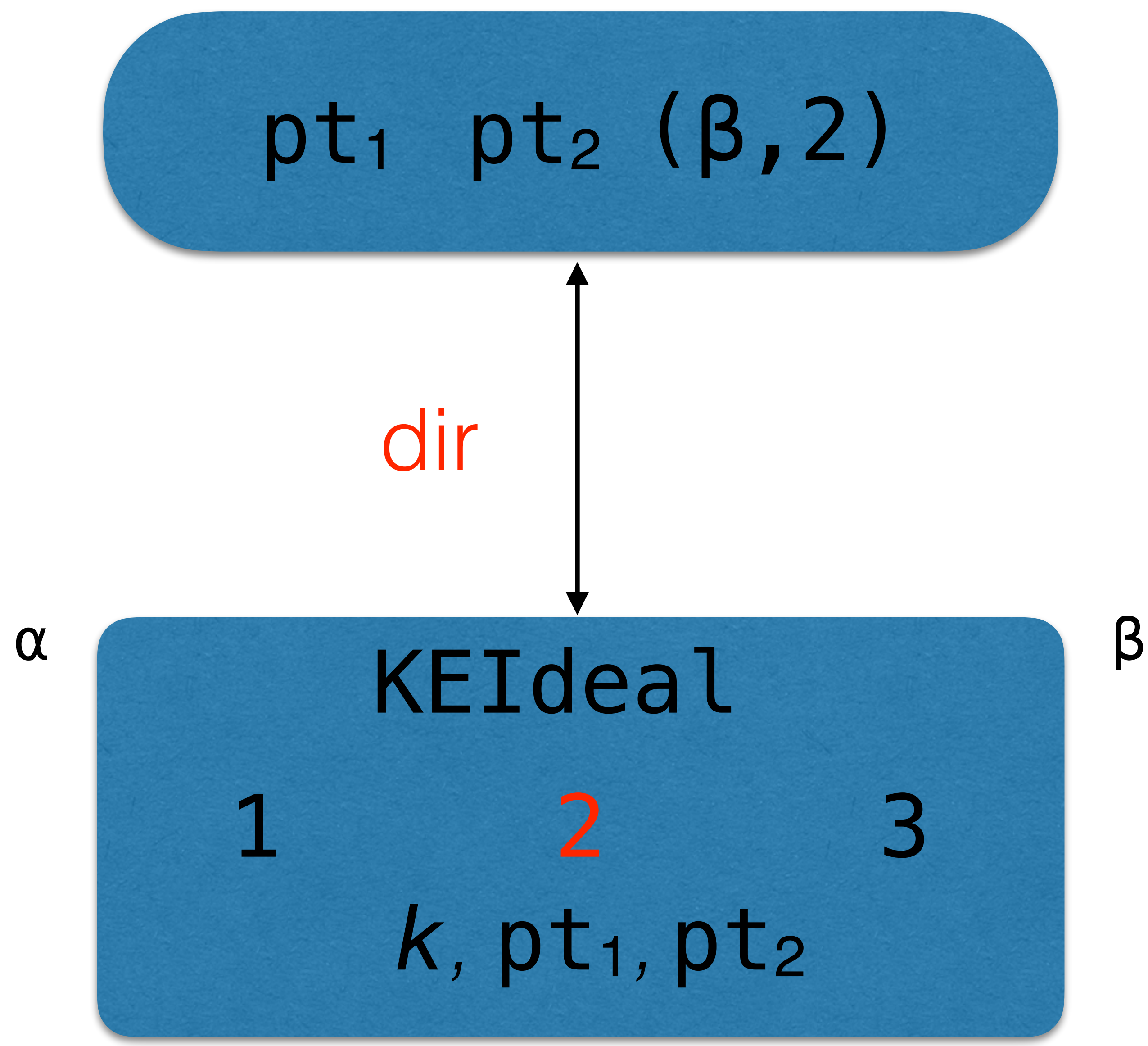

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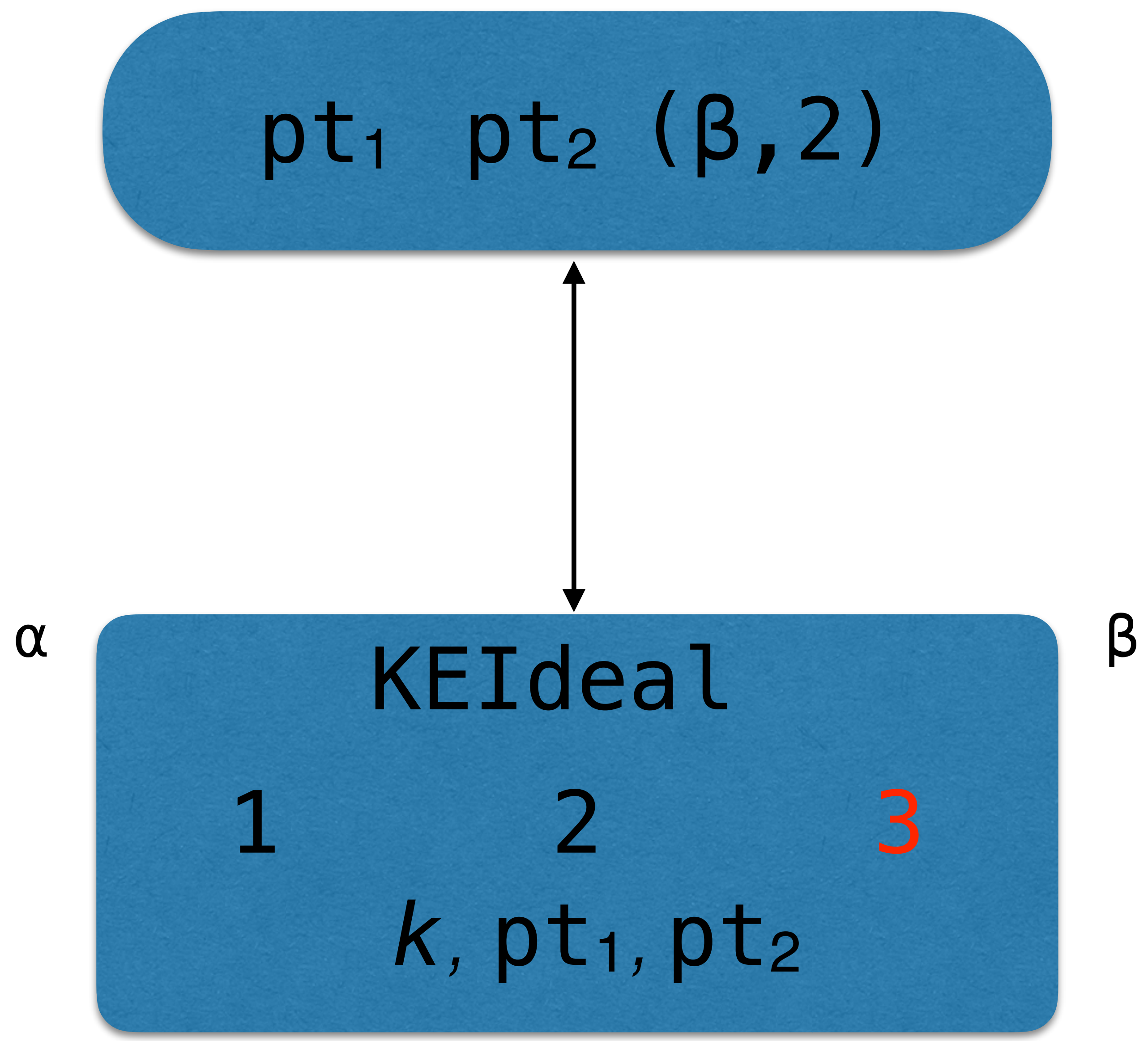

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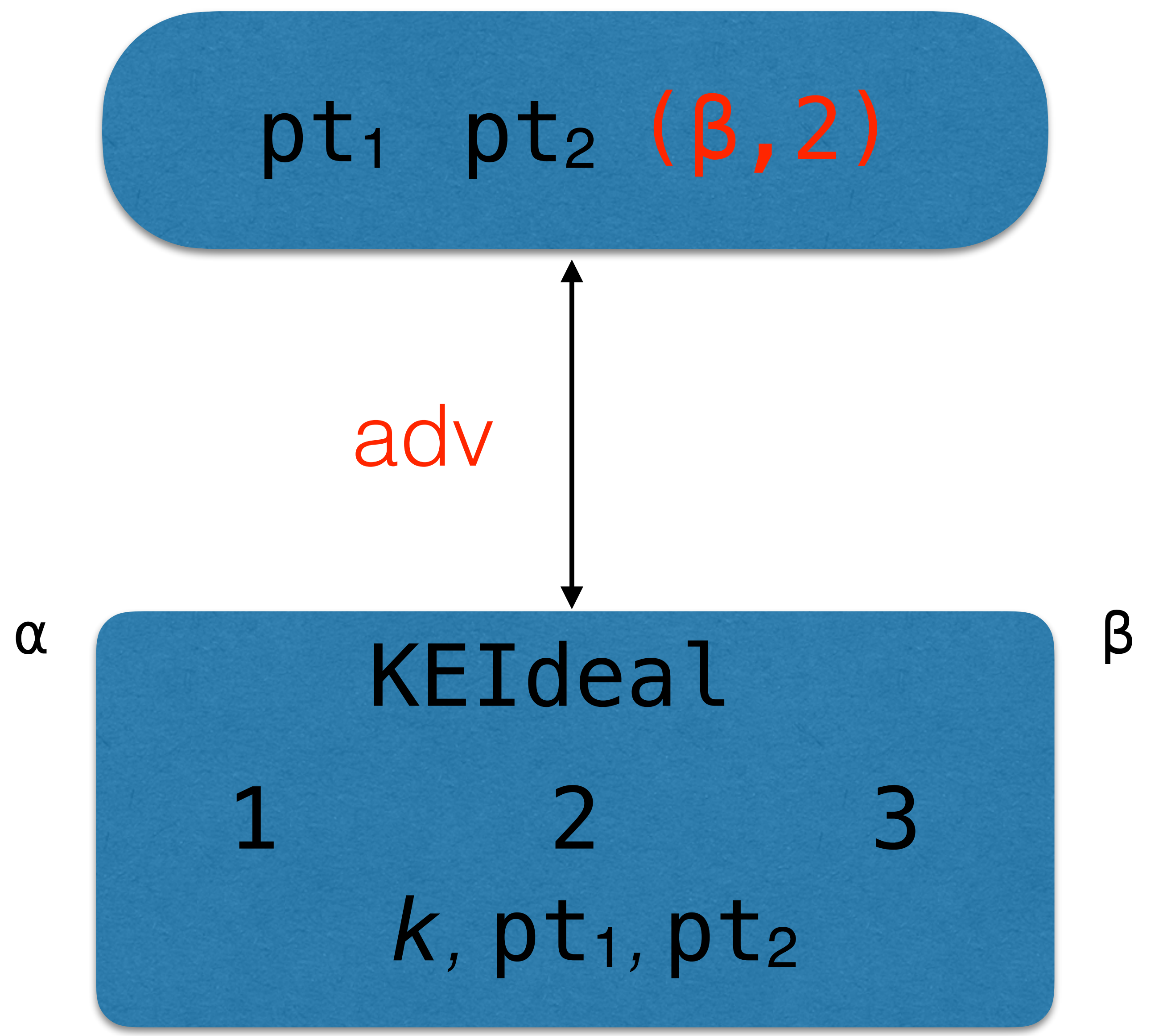

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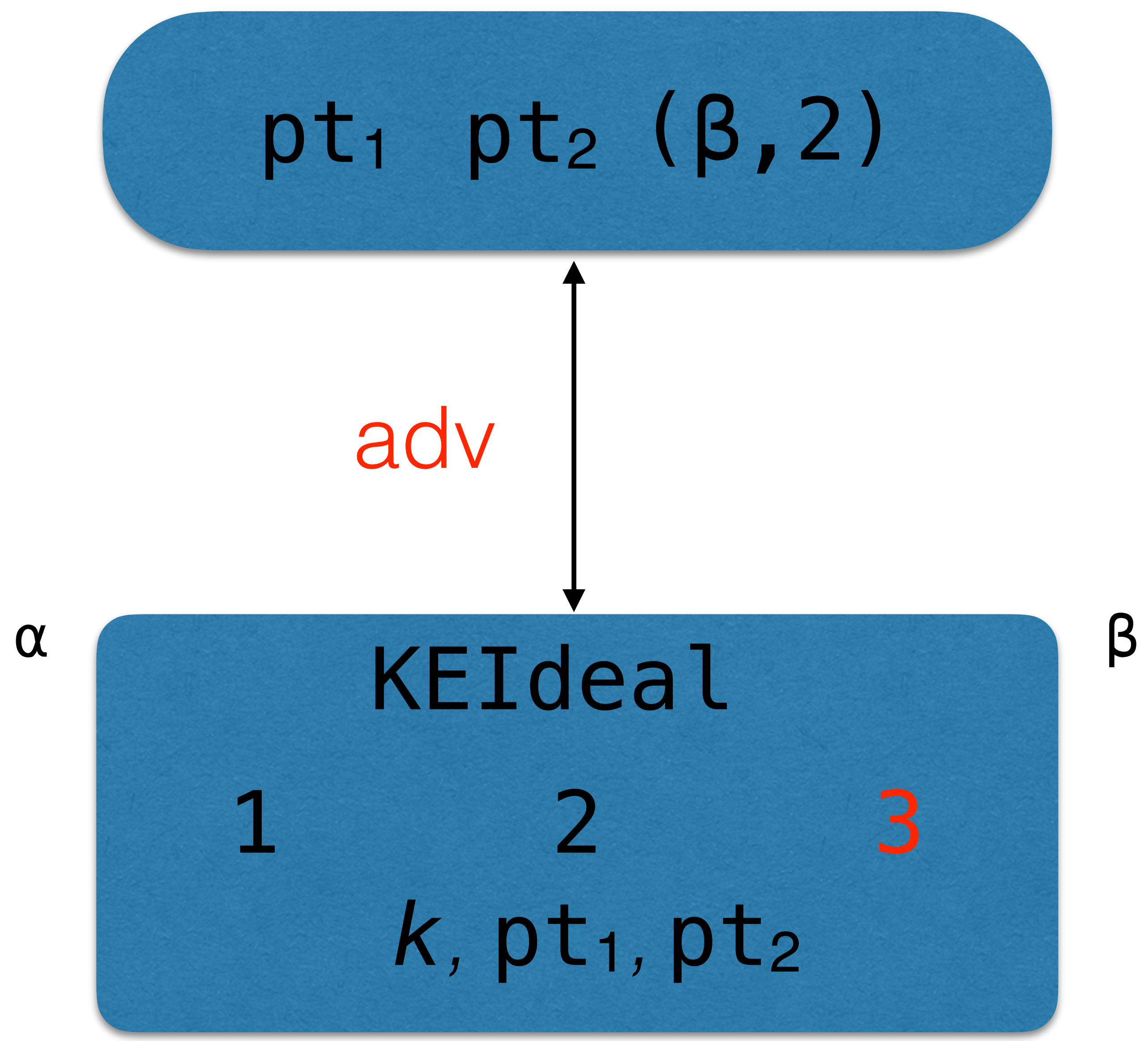
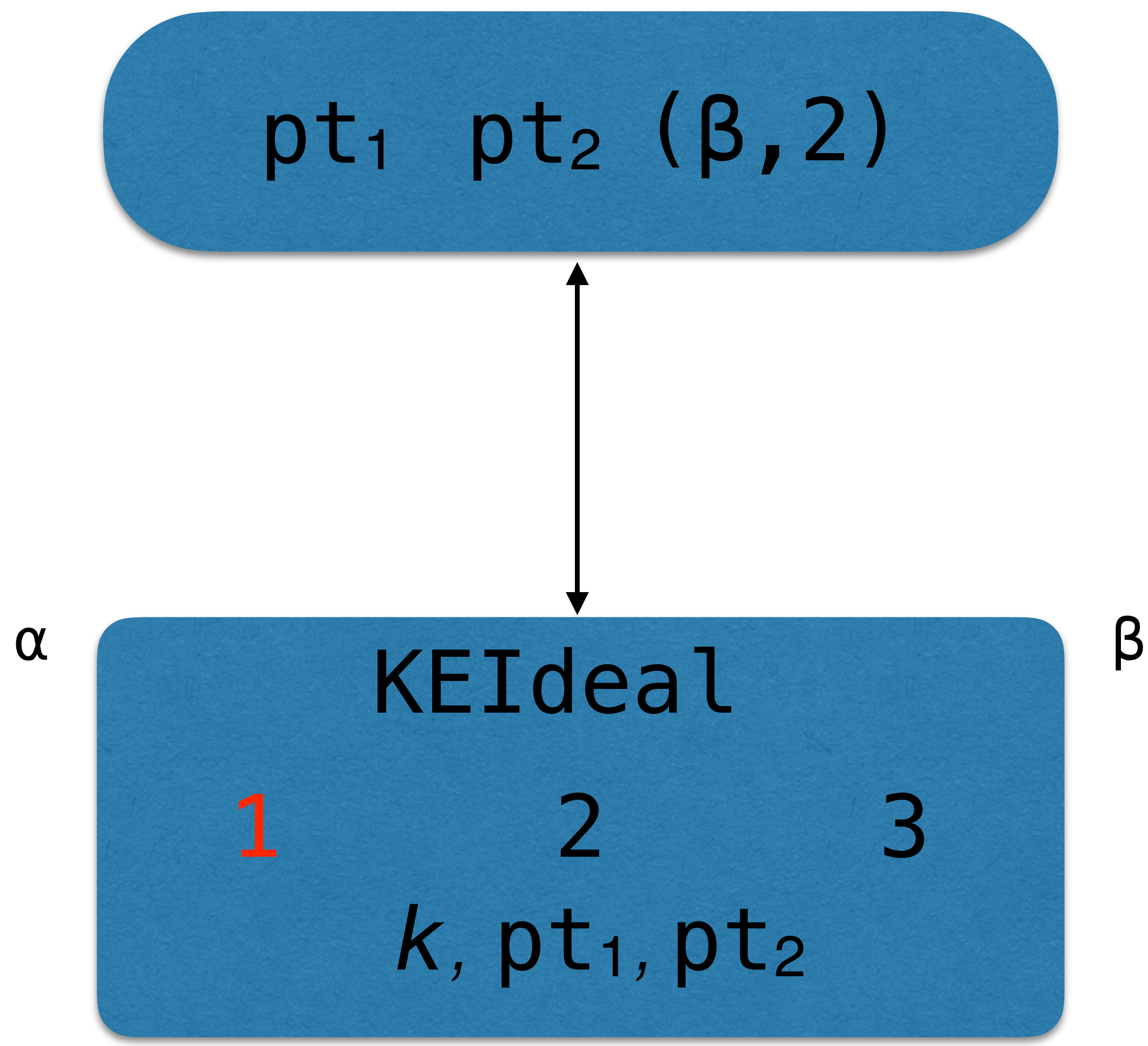
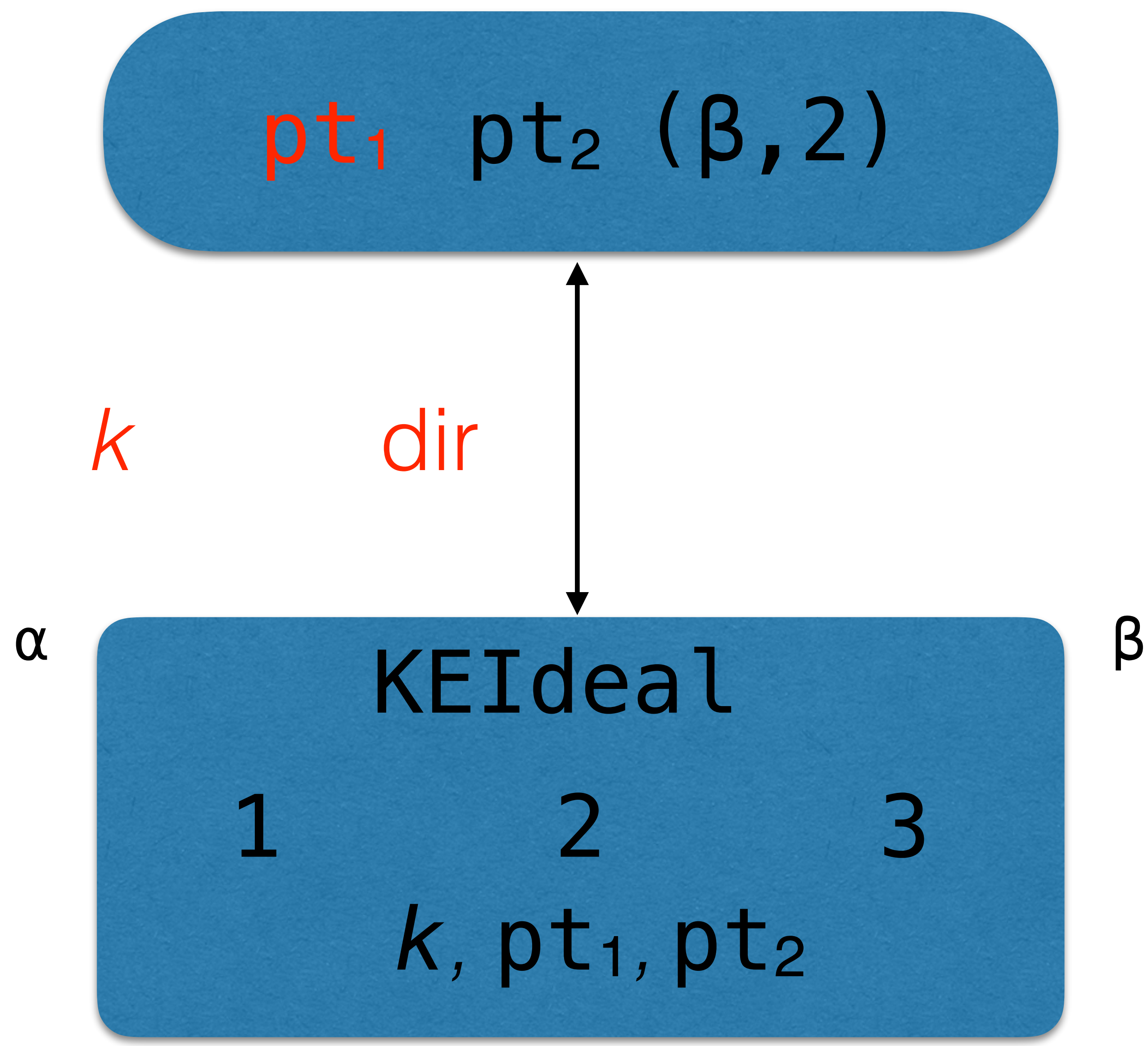


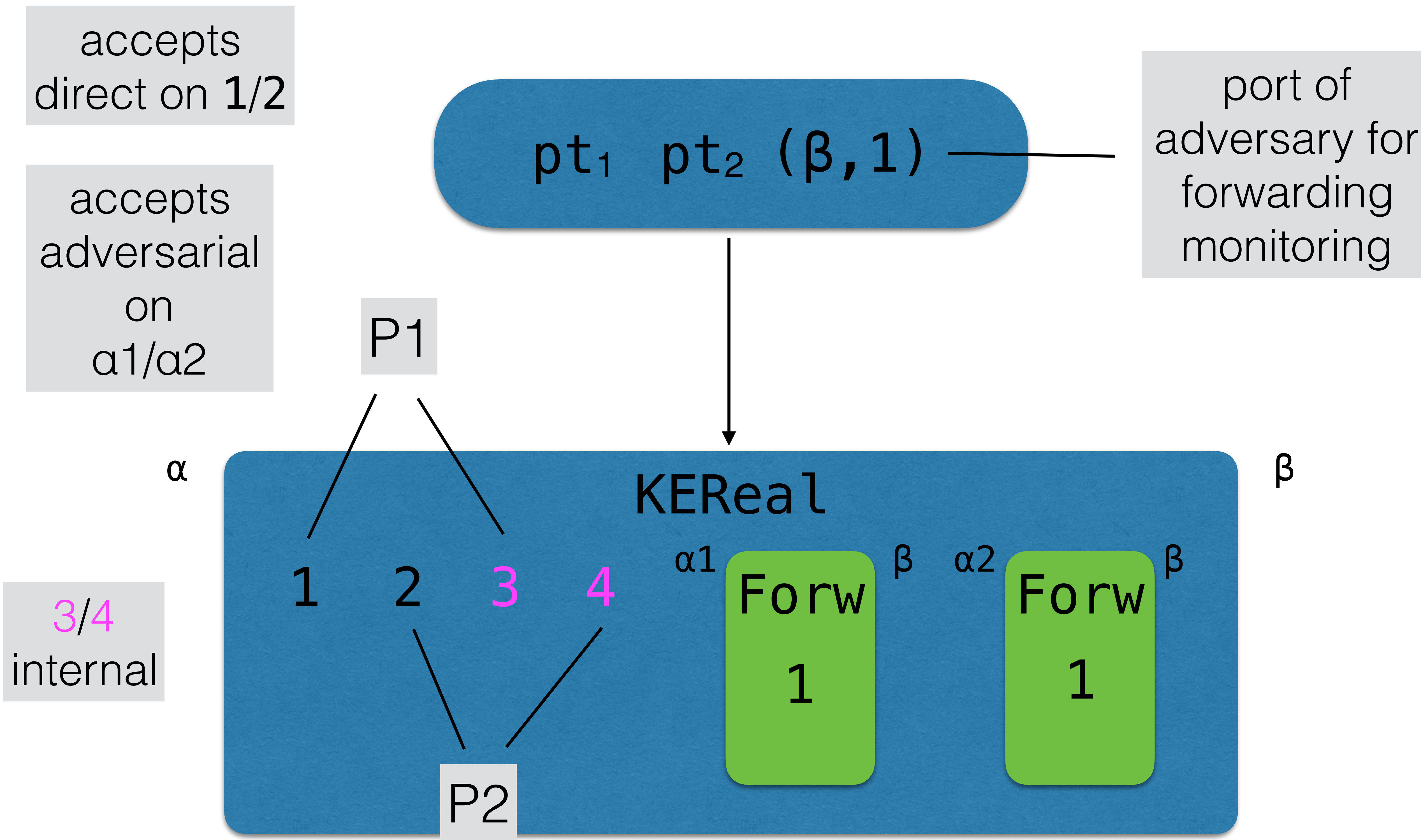
Key Exchange Ideal Functionality



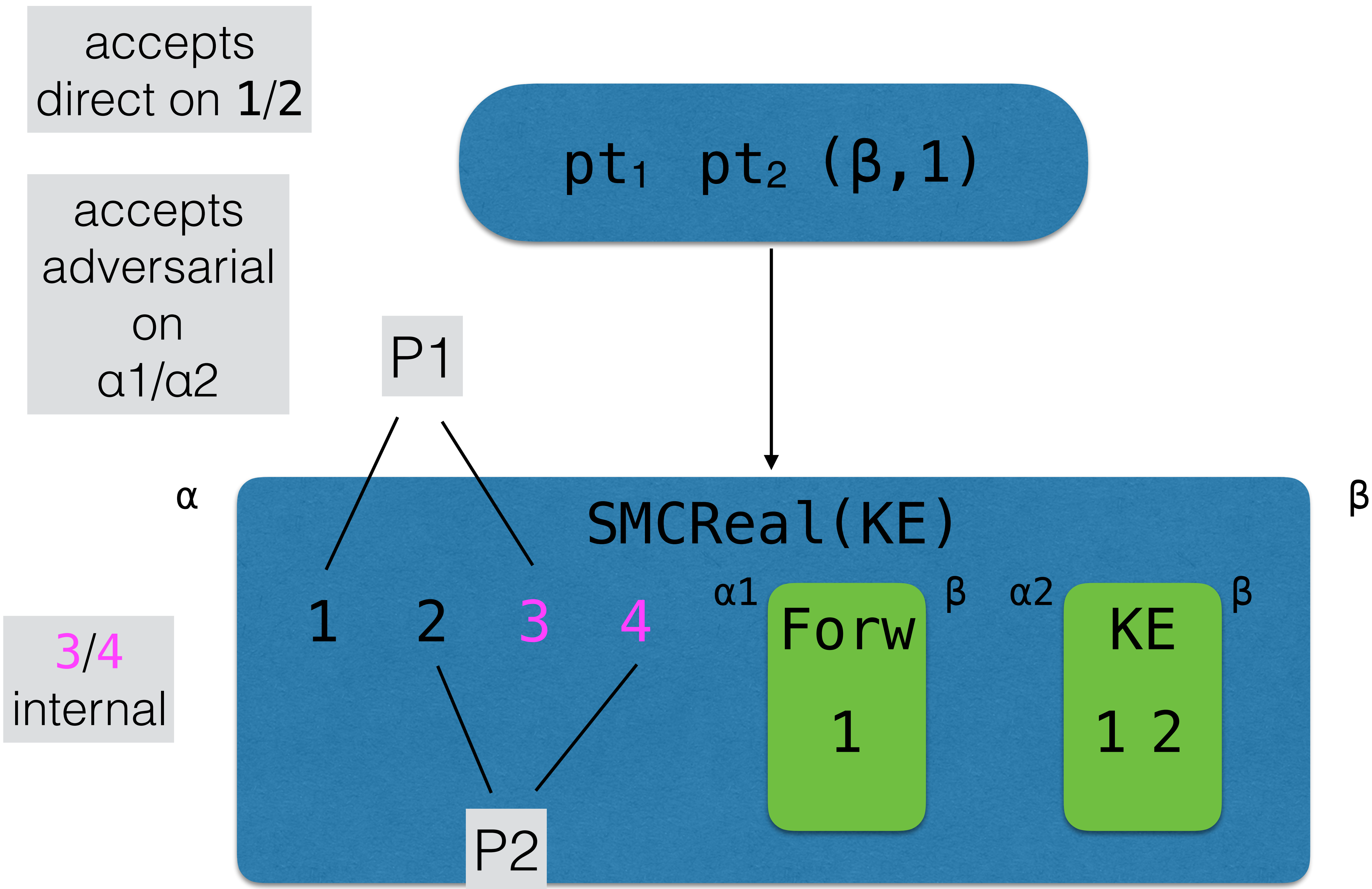
Key Exchange Ideal Functionality



Key Exchange Real Functionality

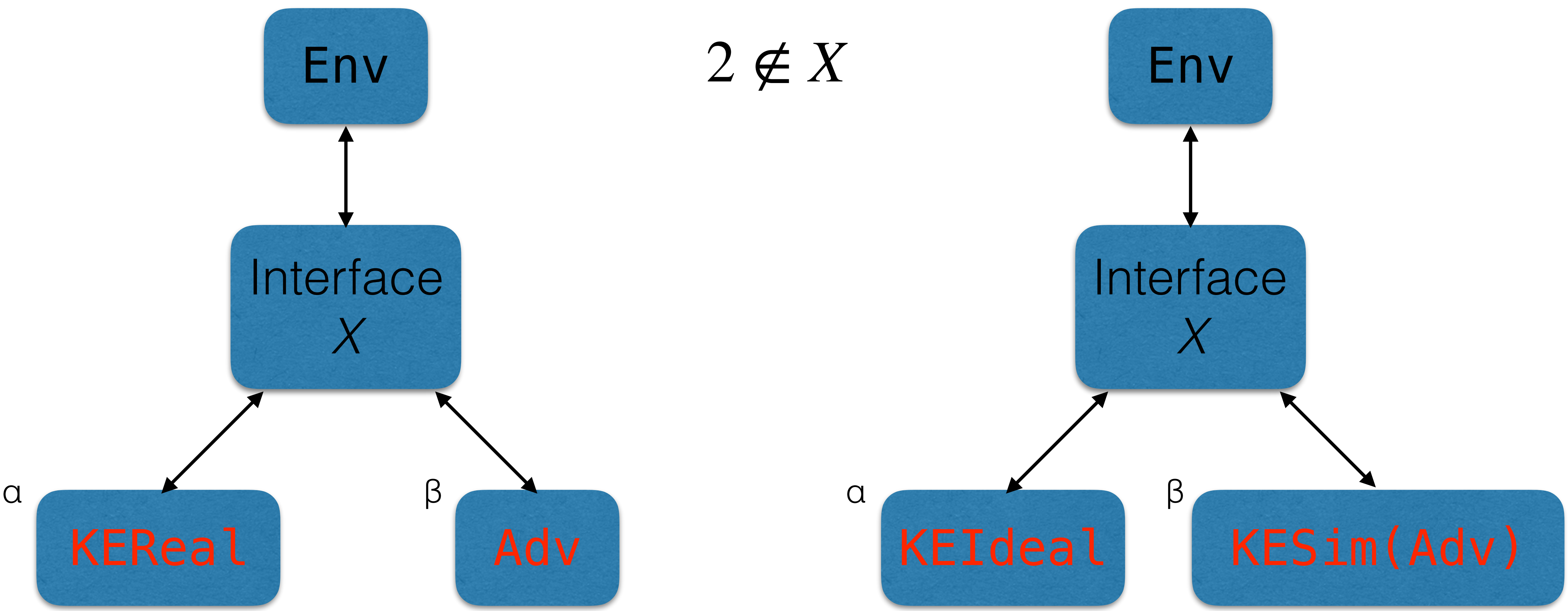


SMC Real Functionality

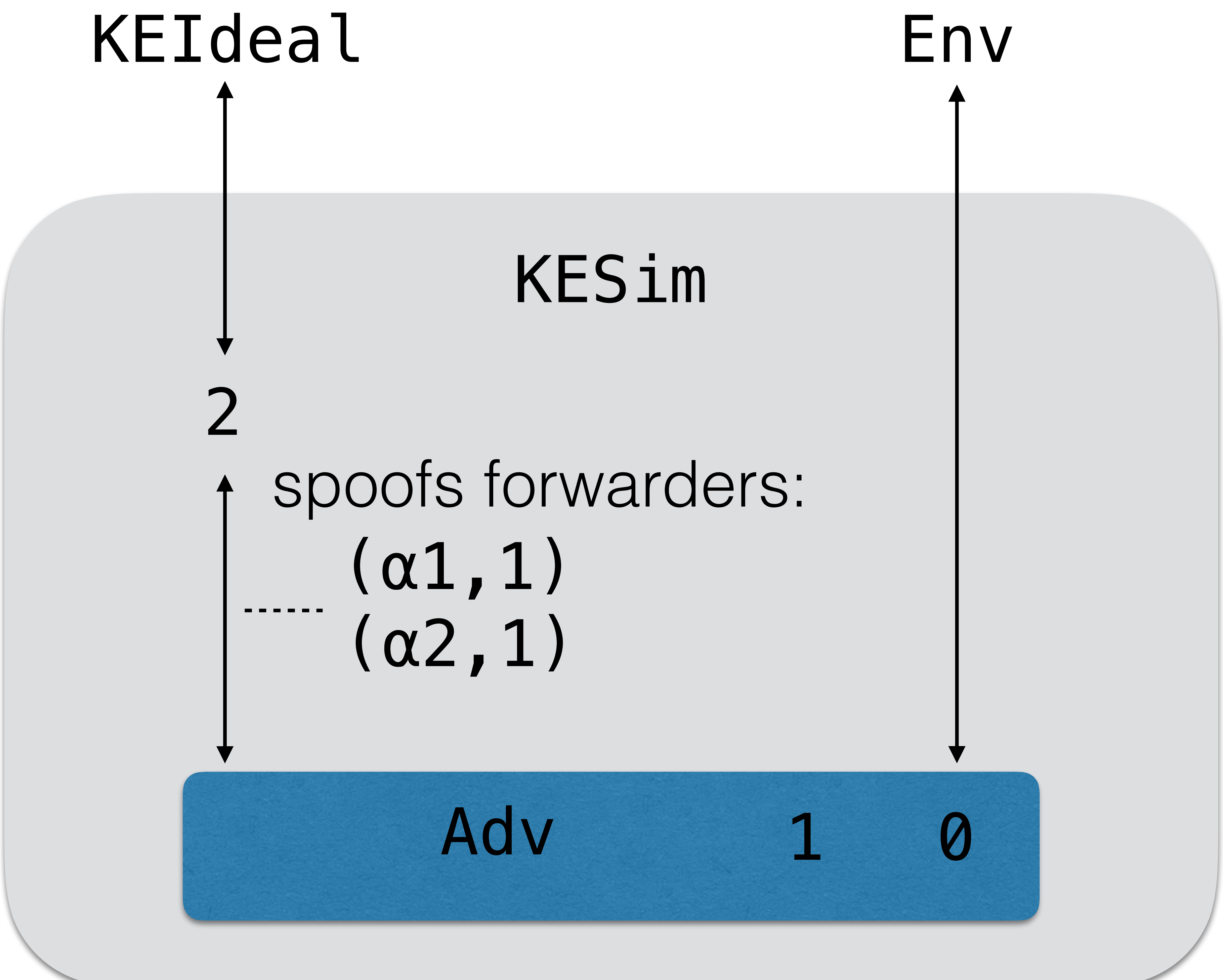


Key Exchange Security

To prove the security of key exchange, we must formulate a simulator, **`KESim`**, and connect the real and ideal games via a sequence of intermediate games:



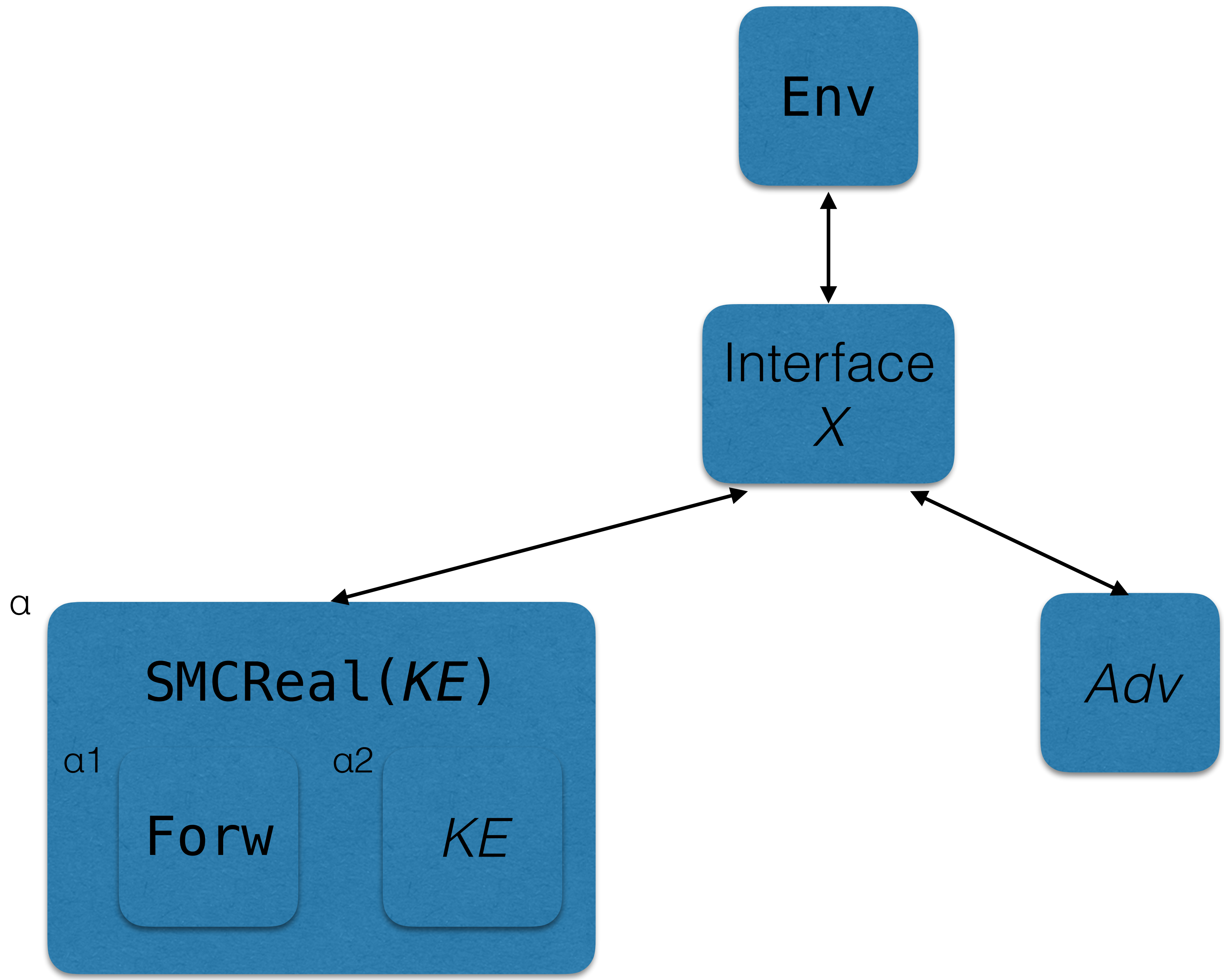
Key Exchange Simulator



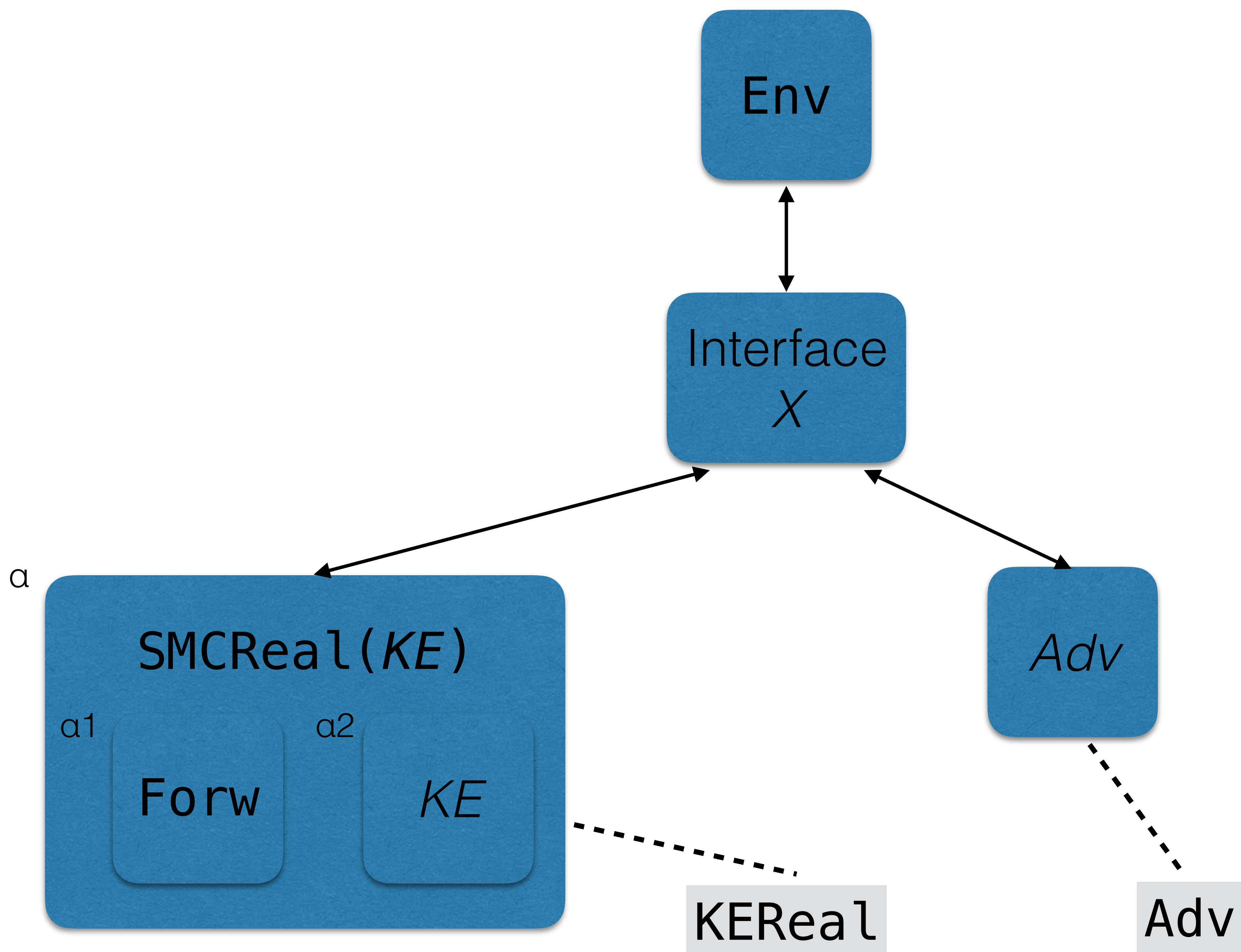
Key Exchange Sequence of Games

- Use EasyCrypt's eager/lazy sampling to move choices of random exponents to beginning of game
- Reduce to Decisional Diffie-Hellman (DDH) assumption
  - Constructed DDH adversary parameterized by `Env` and `Adv`
- Now the agreed upon key is `g` ^ q_3, for a random q_3
- Use eager/lazy sampling to delay generation of exponents
- Connect this hybrid game with ideal game

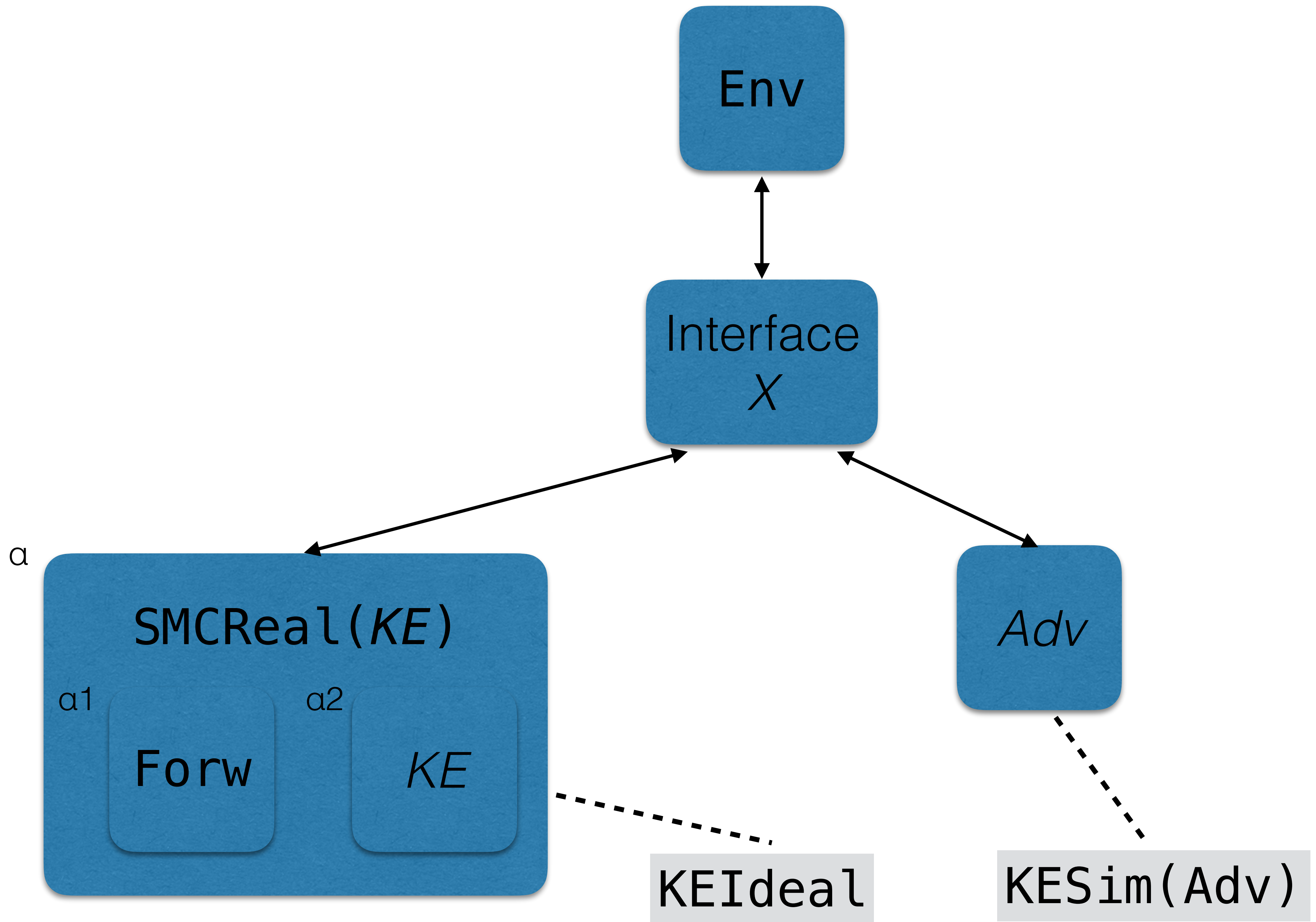
Instance of Composition Theorem



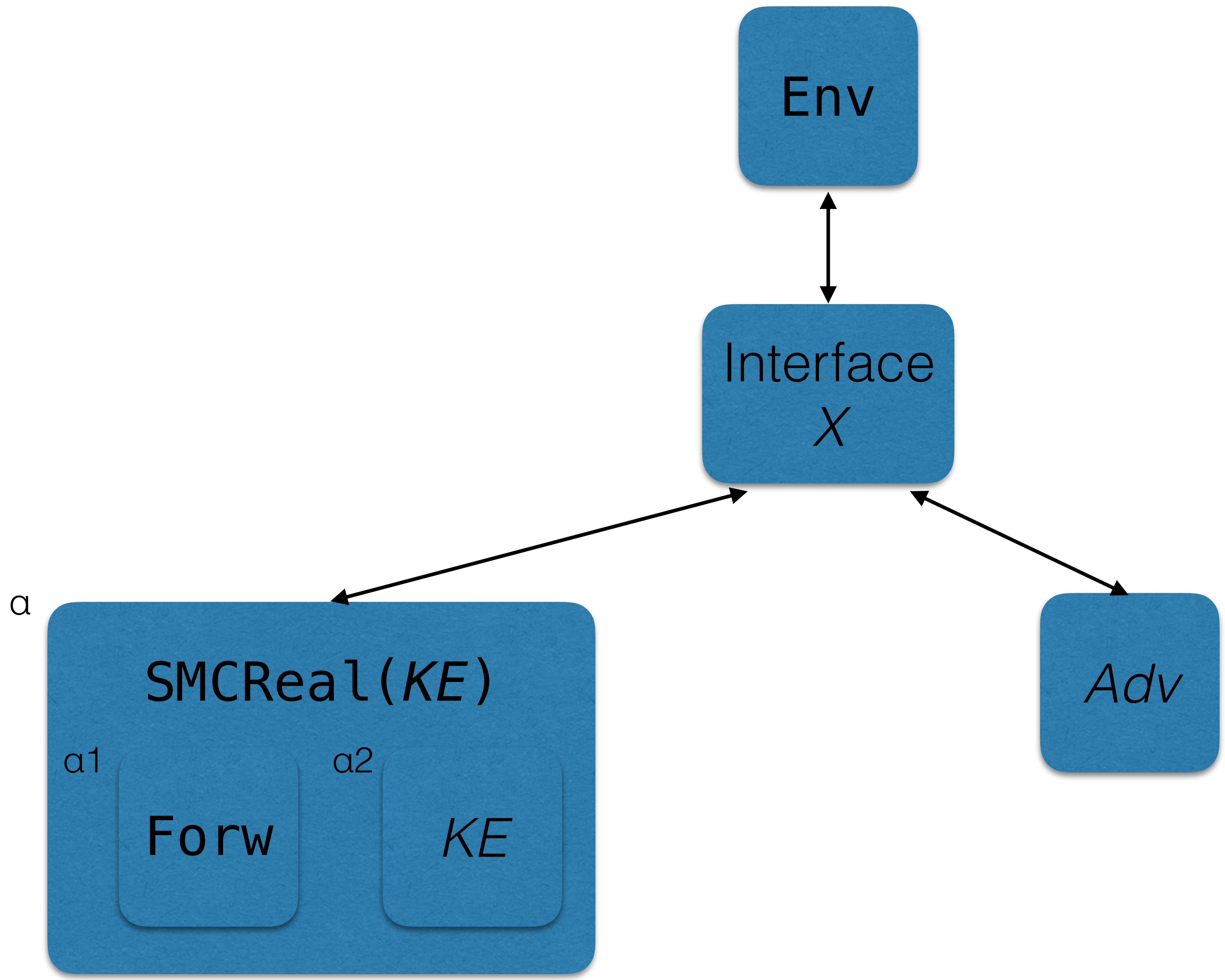
Instance of Composition Theorem



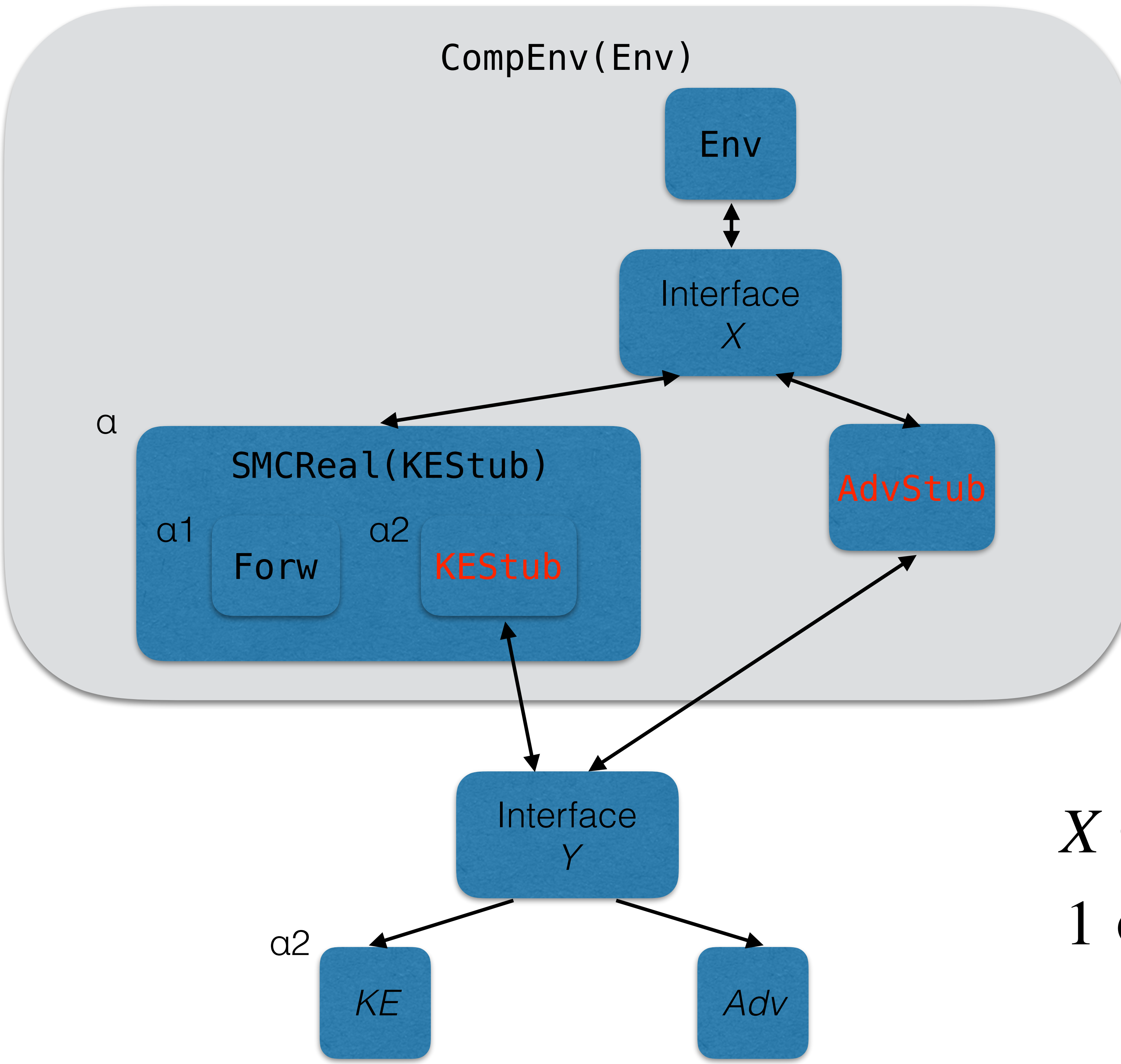
Instance Composition Theorem



Bridging Lemma for Composition Theorem



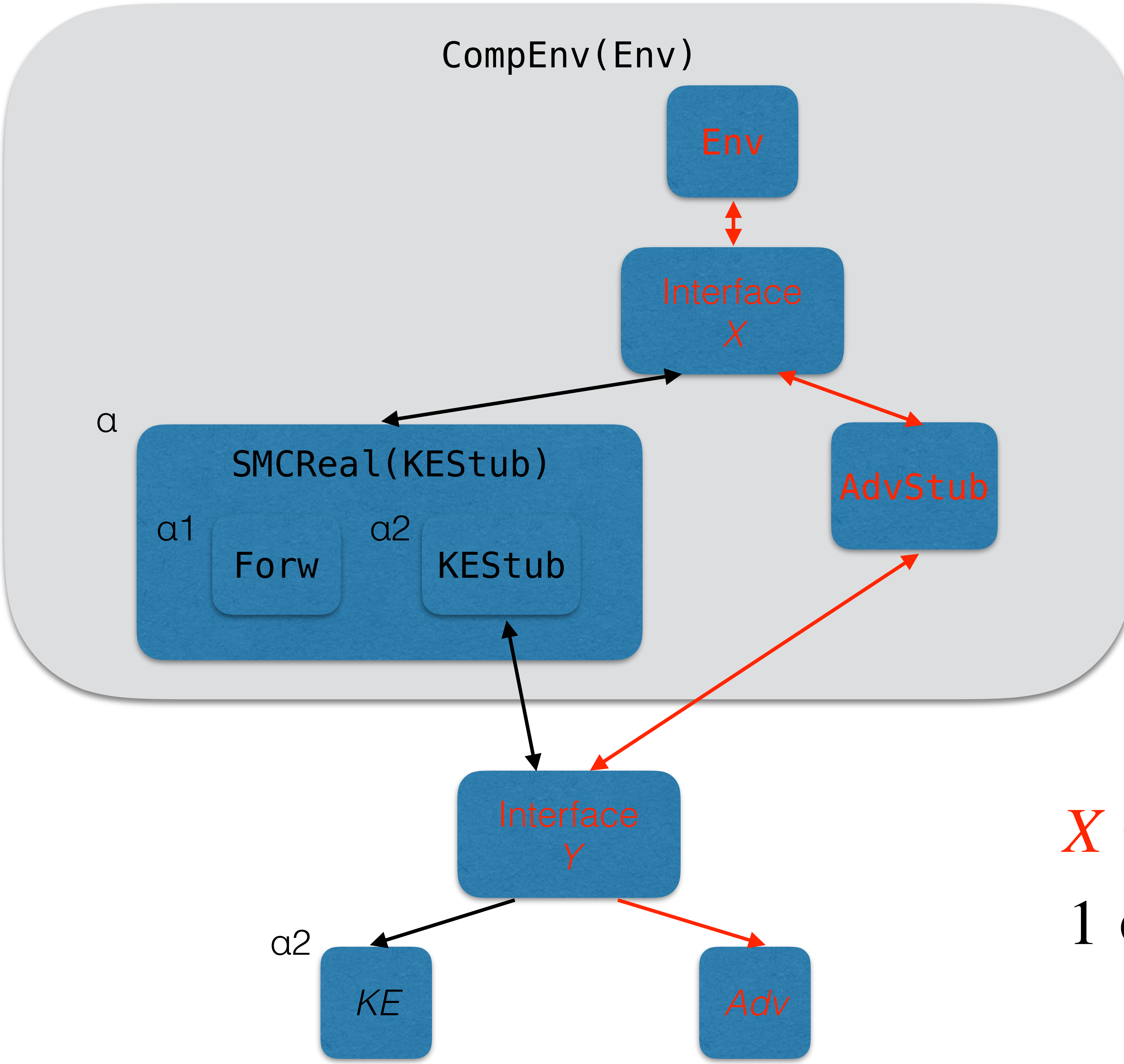
Bridging Lemma for Composition Theorem



$X \subseteq Y$

$1 \in Y$

Bridging Lemma for Composition Theorem



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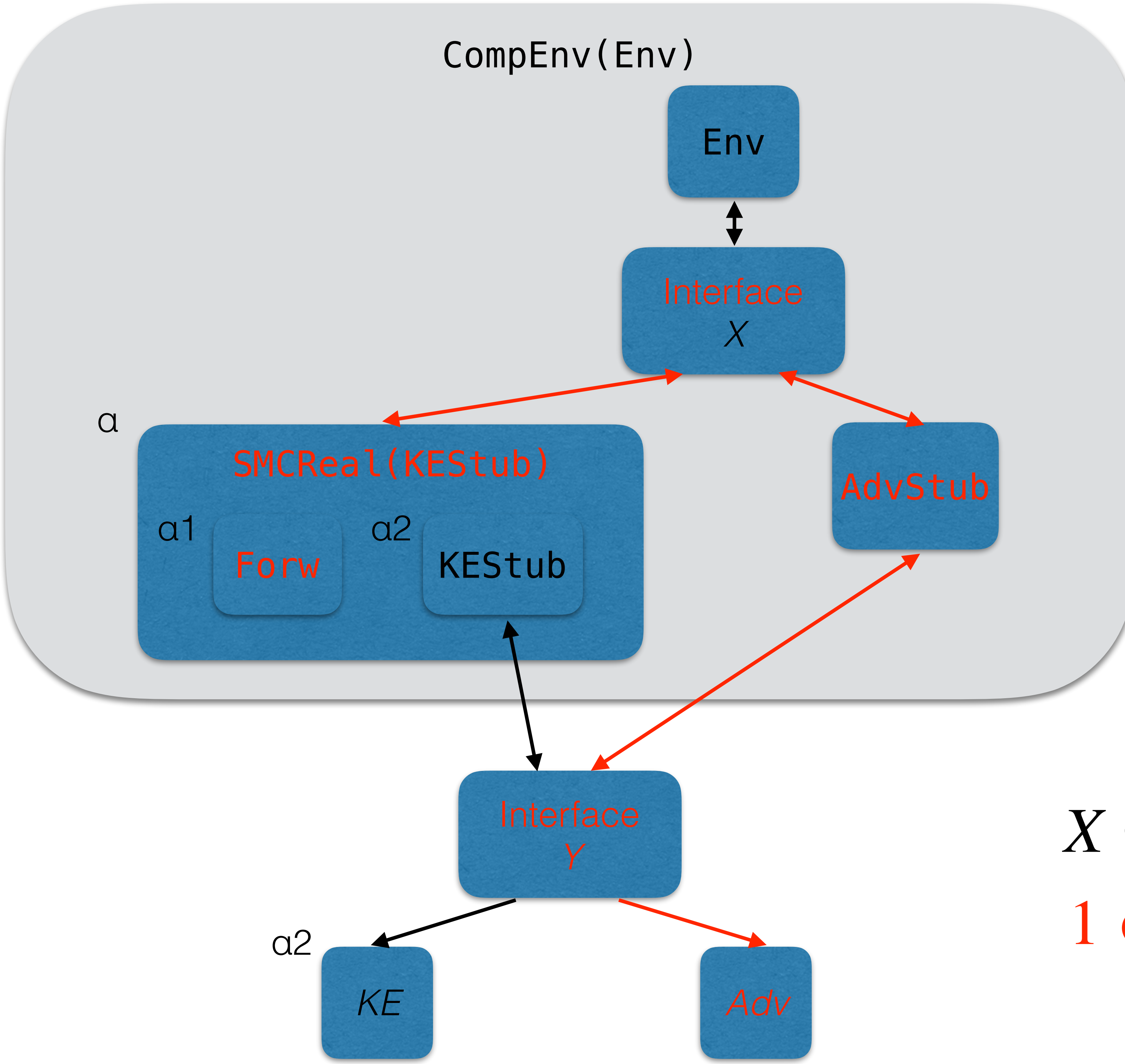

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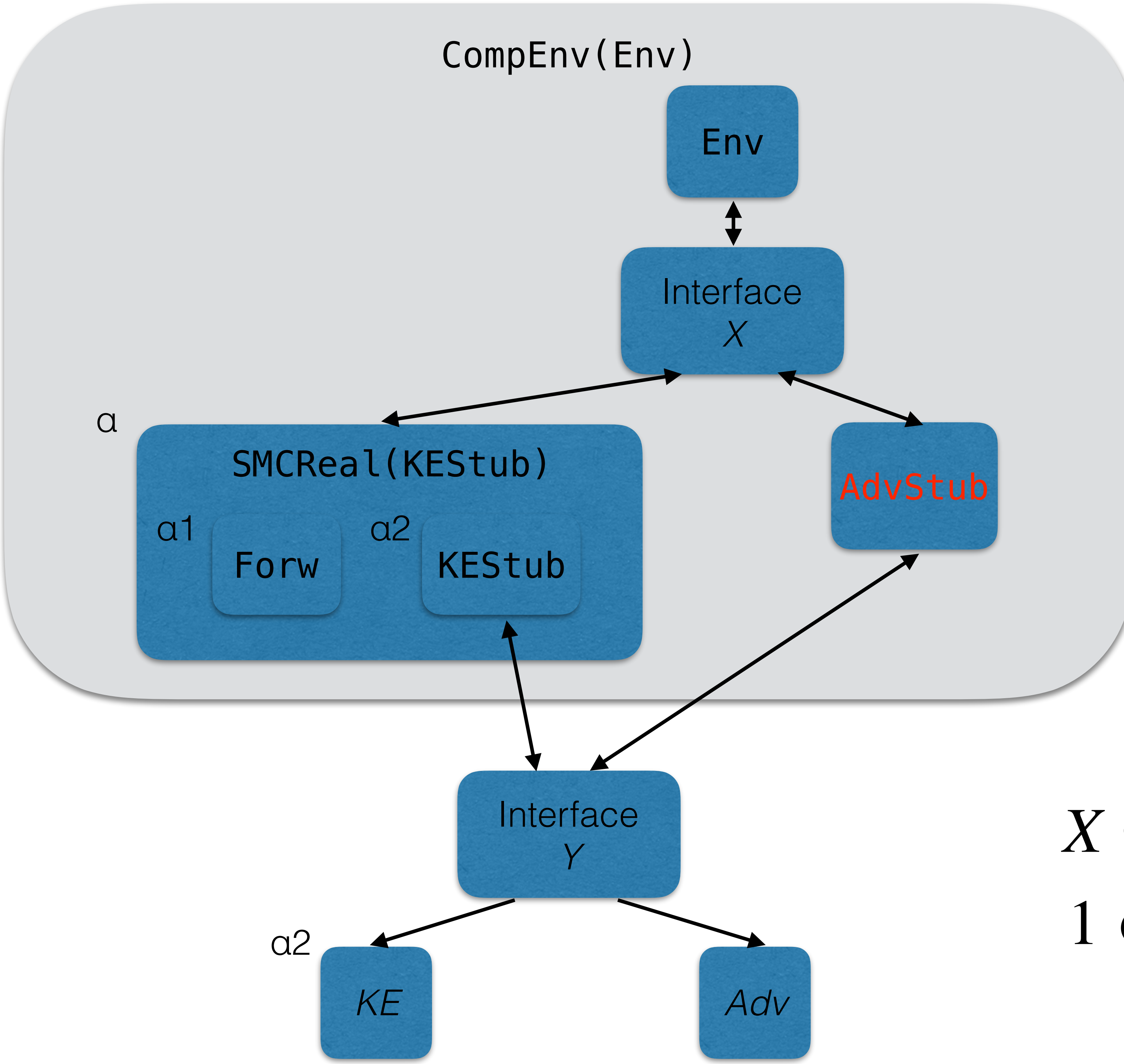

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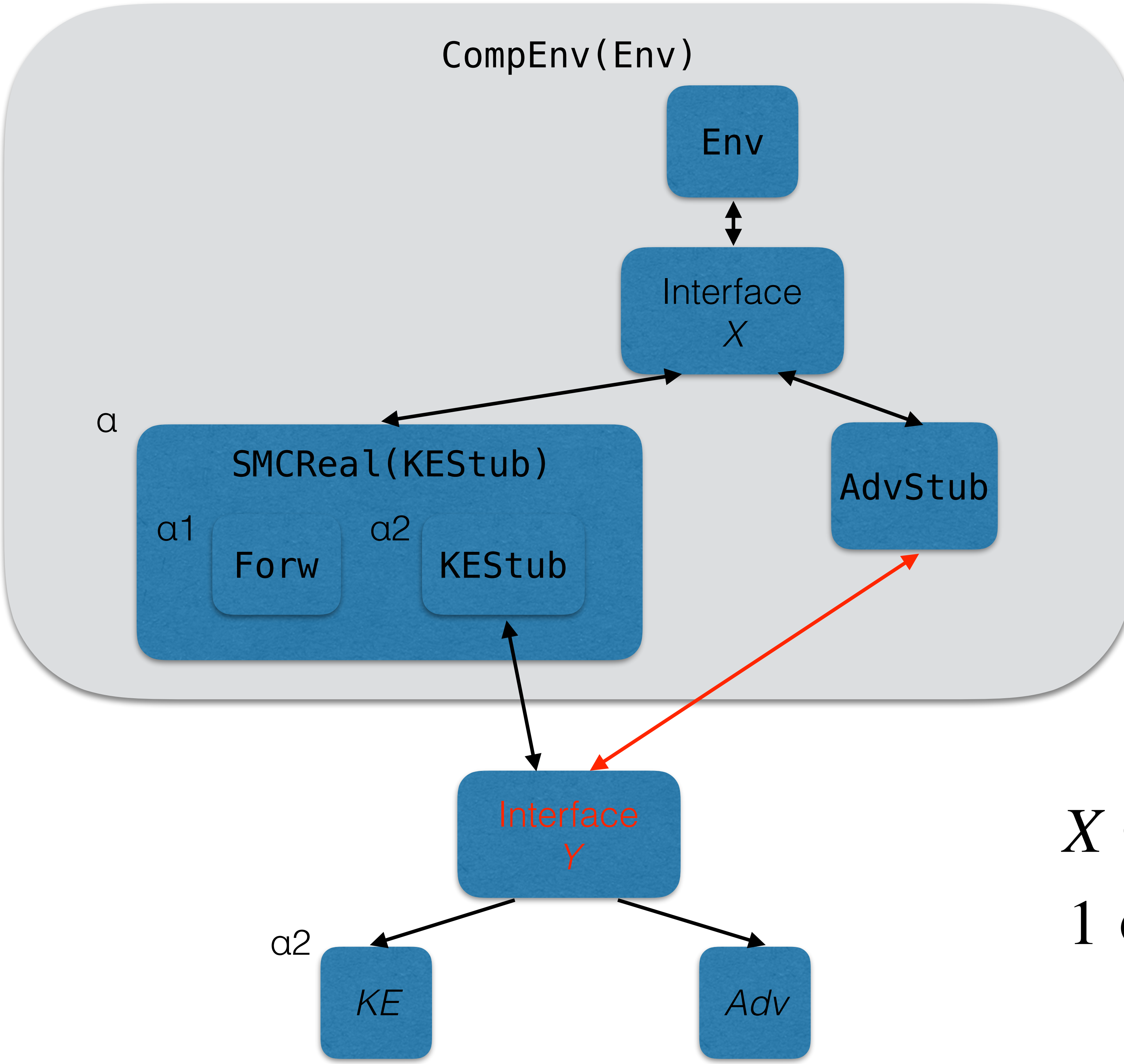

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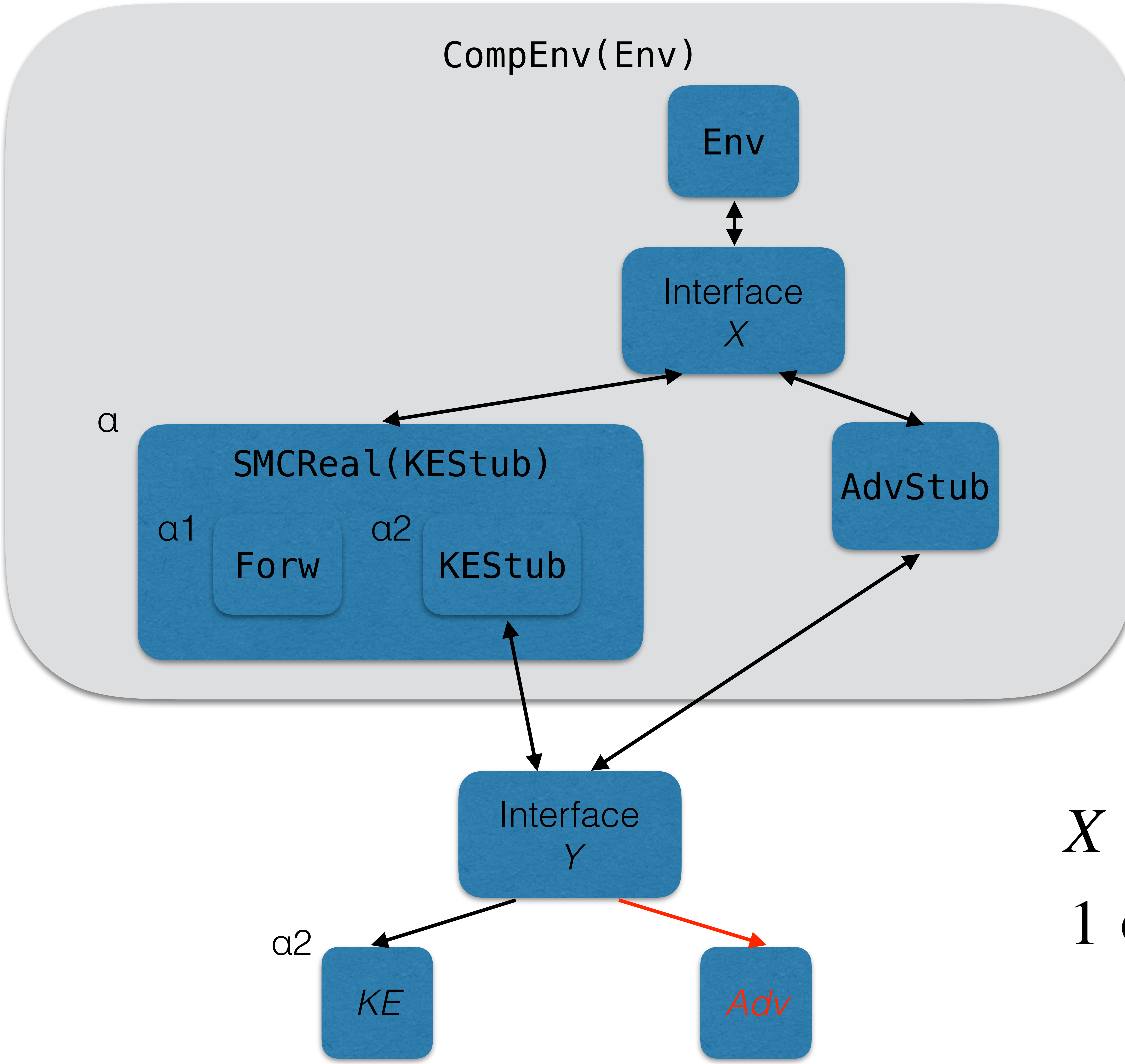
Bridging Lemma for Composition Theorem



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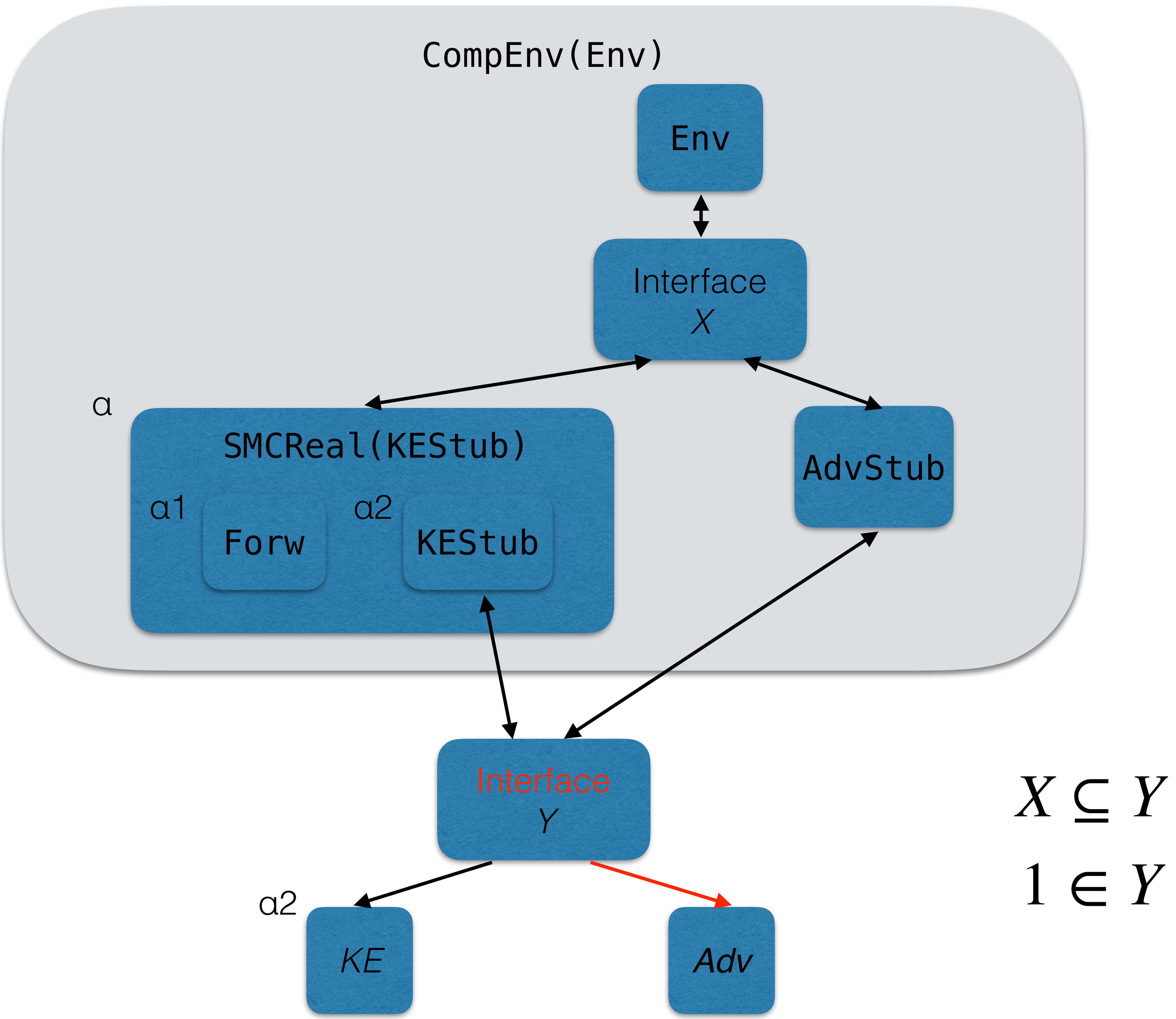
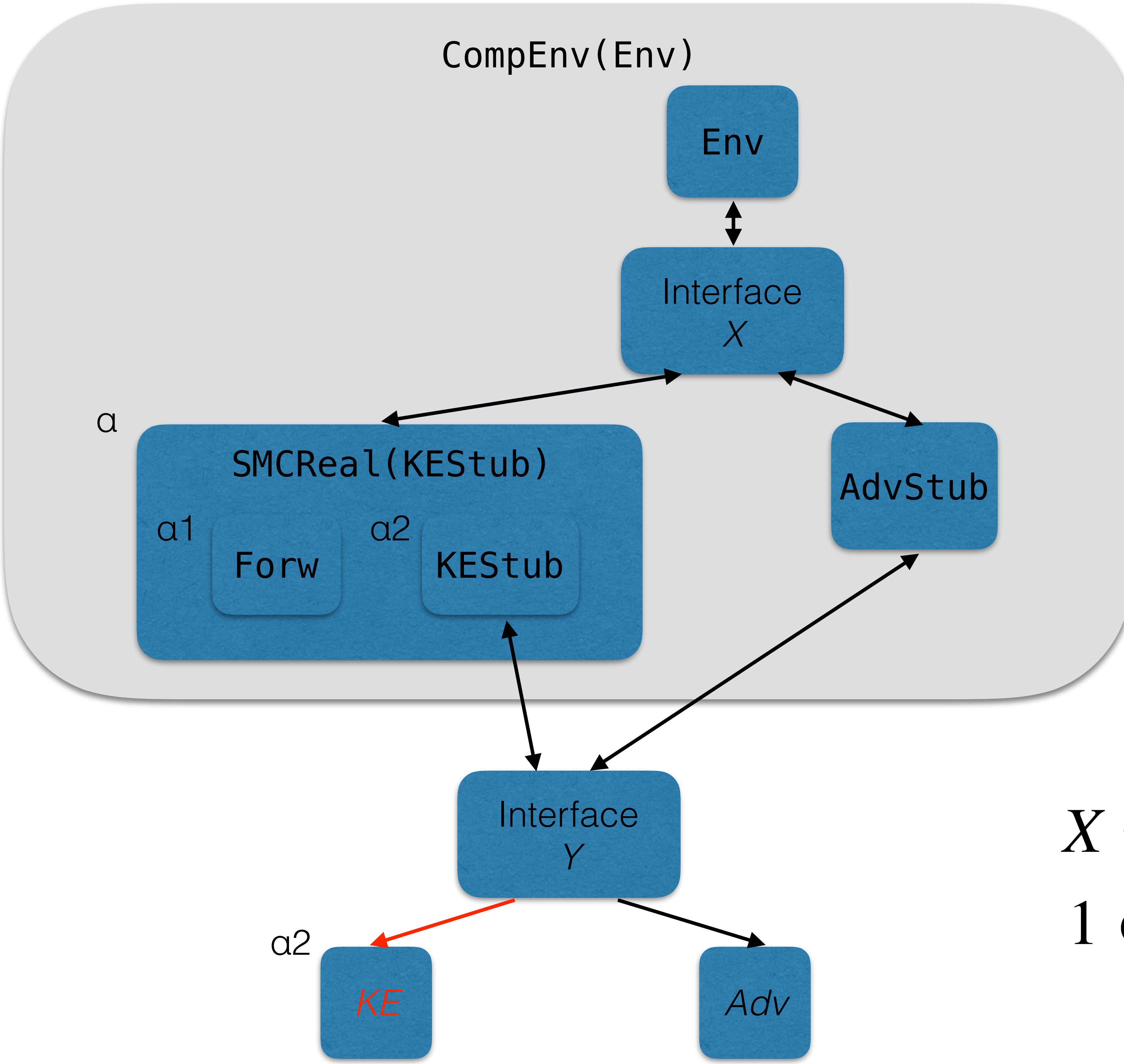


Bridging Lemma for Composition Theorem



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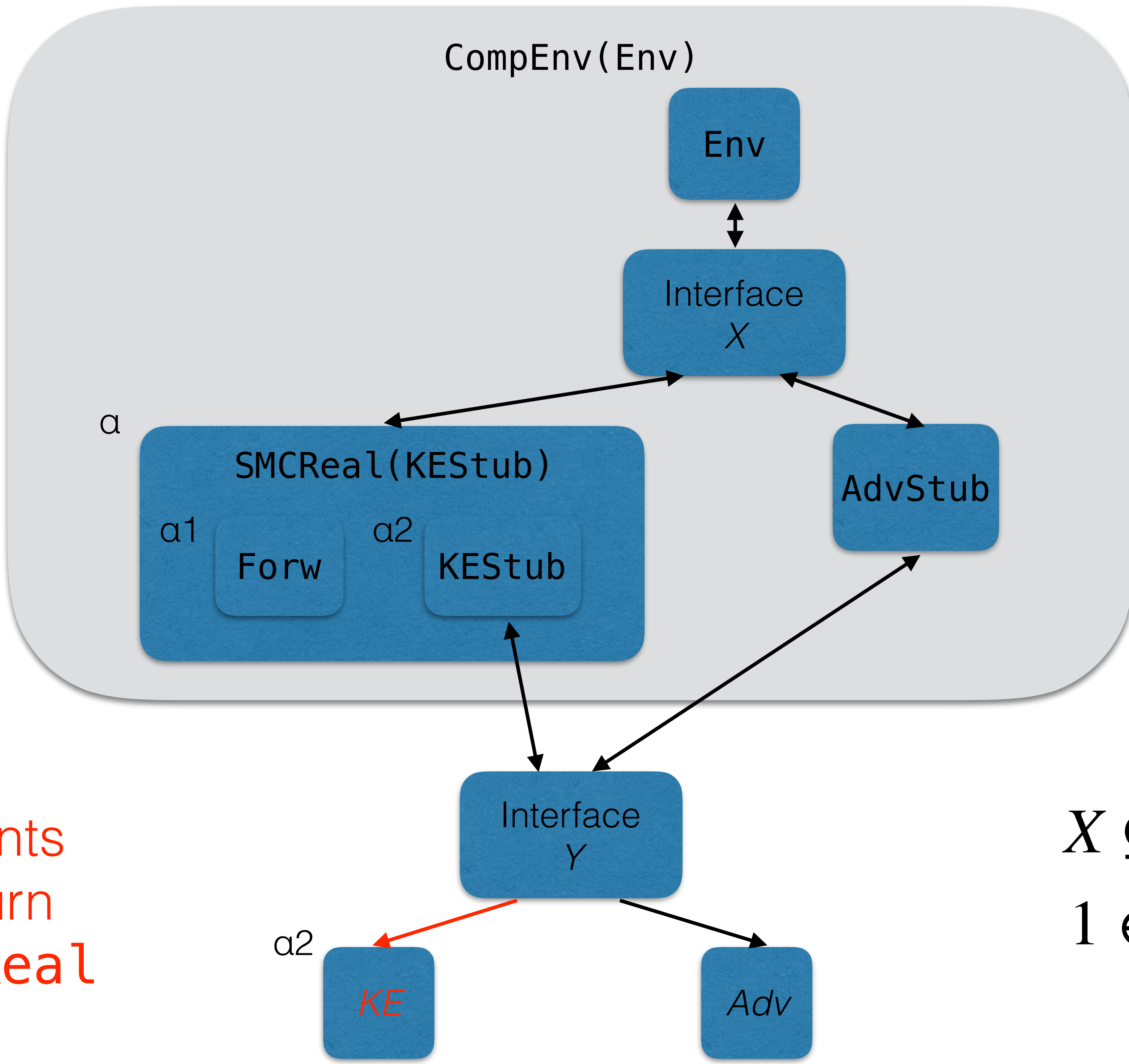
Bridging Lemma for Composition Theorem

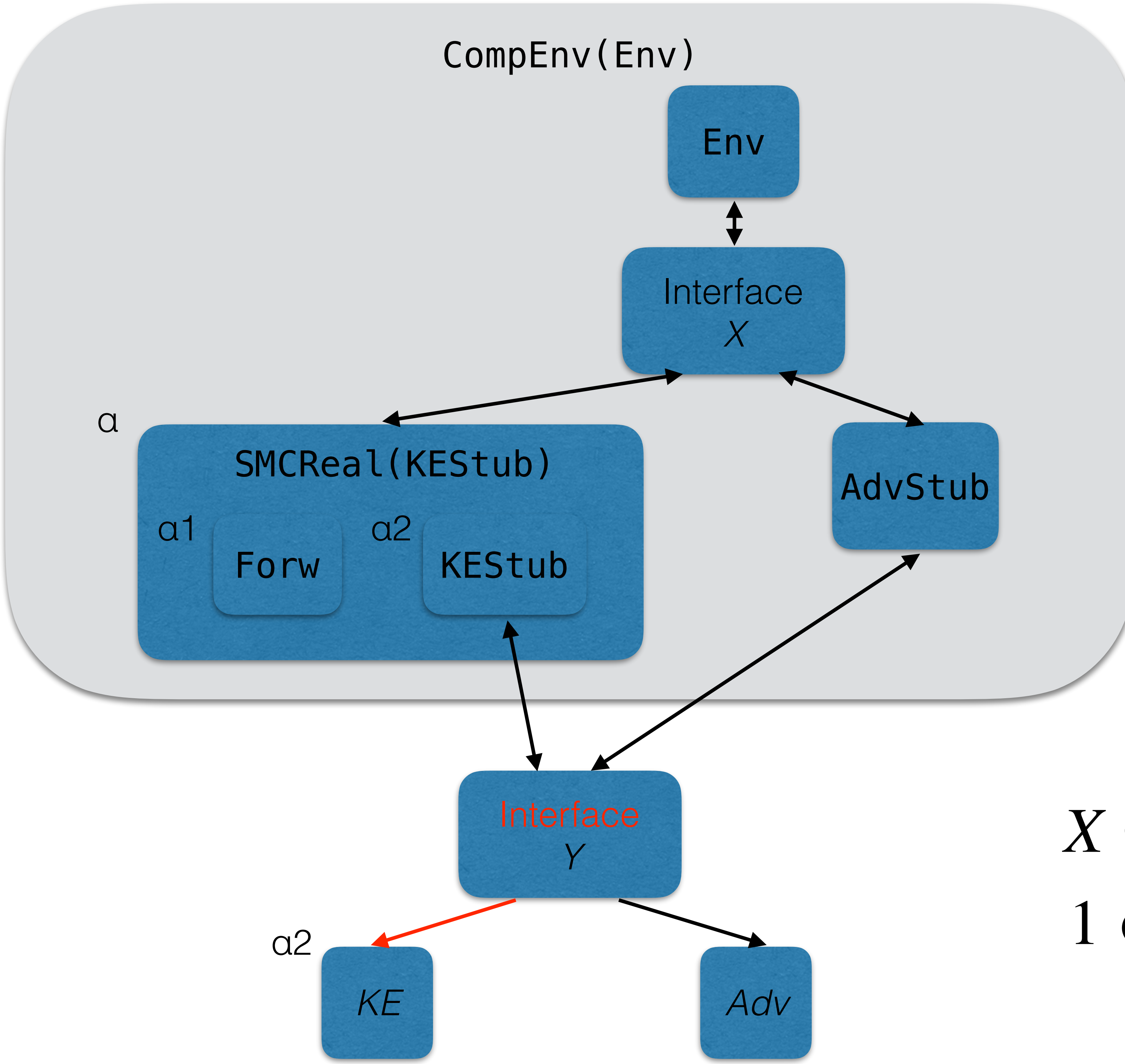


KE wants
to return
to **SMCReal**

$X \subseteq Y$

$1 \in Y$

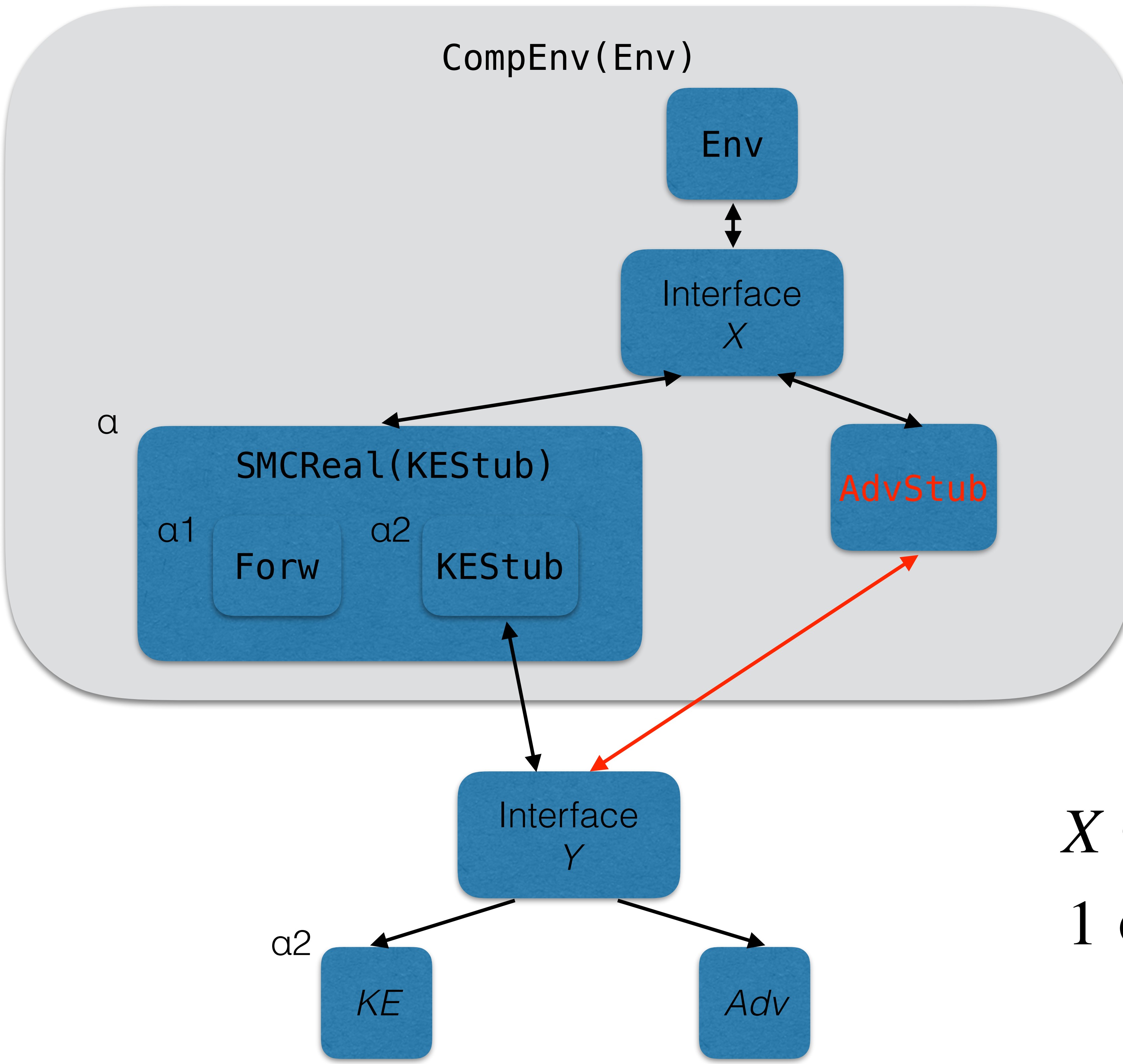
Bridging Lemma for Composition Theorem



$$X \subseteq Y$$

$$1 \in Y$$

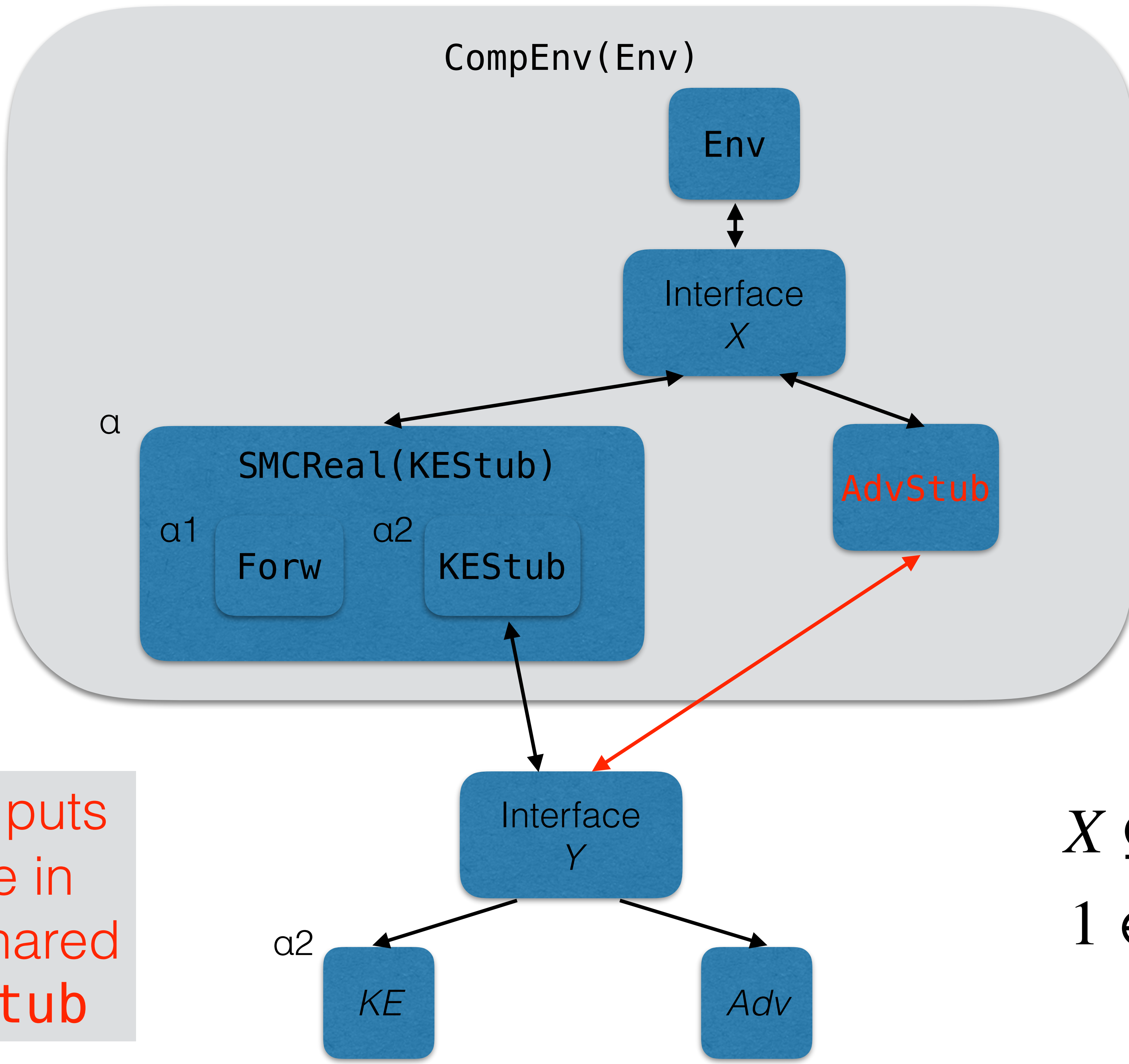
Bridging Lemma for Composition Theorem



$X \subseteq Y$

$1 \in Y$

Bridging Lemma for Composition Theorem

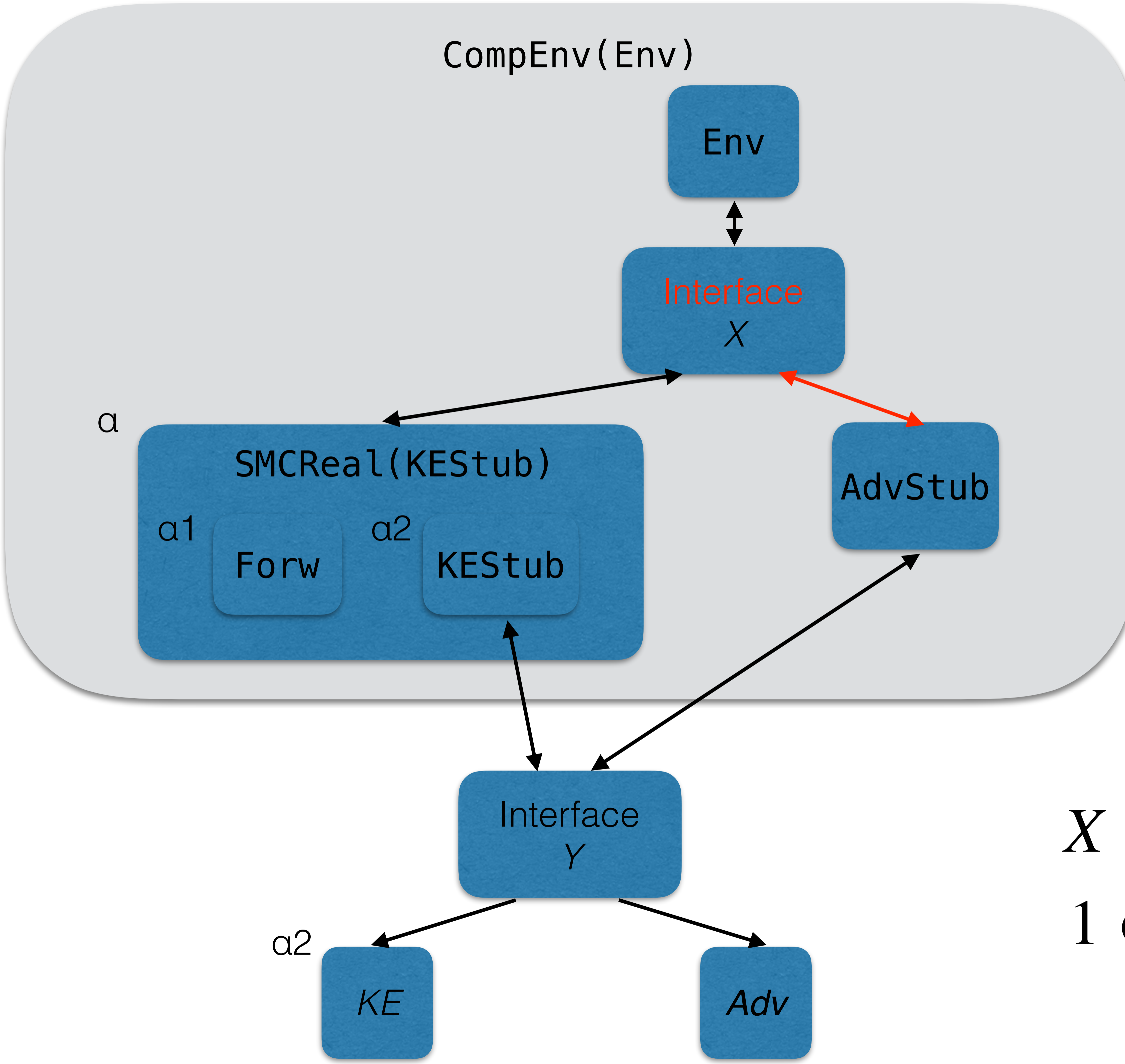


AdvStub puts message in mailbox shared with KEStub

$X \subseteq Y$

$1 \in Y$

Bridging Lemma for Composition Theorem



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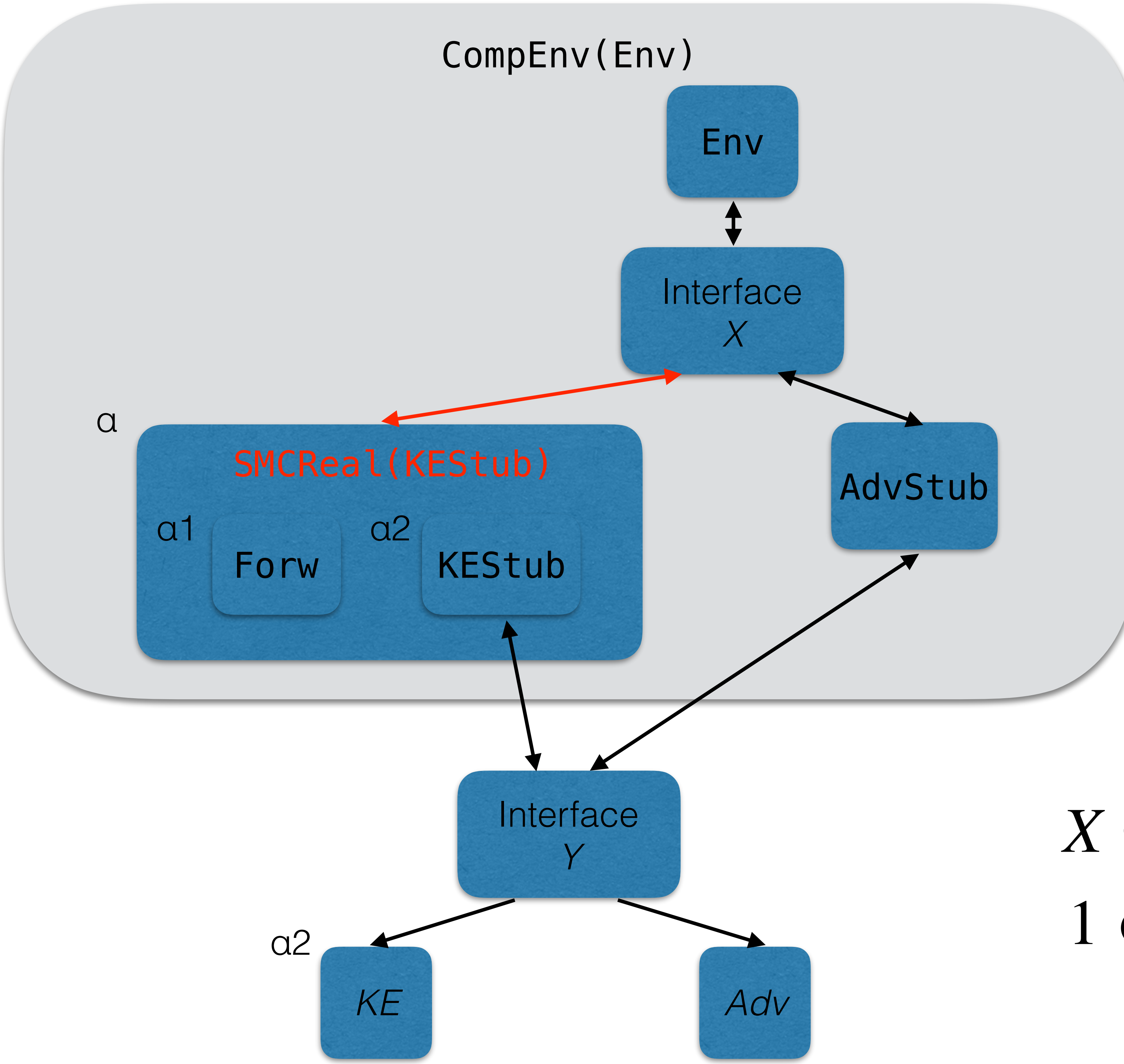
$1 \in Y$

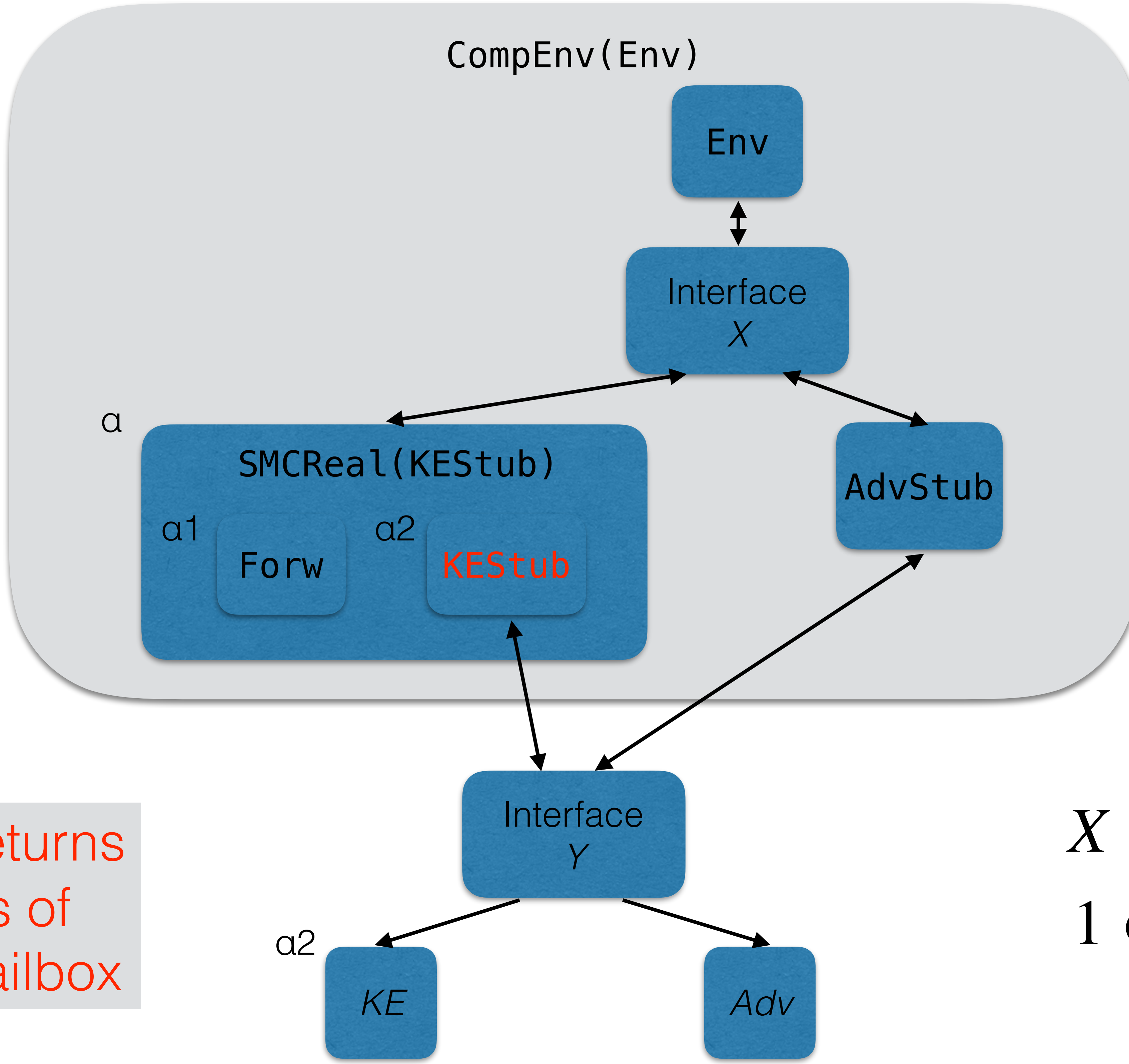
Bridging Lemma for Composition Theorem



$$X \subseteq Y$$

$$1 \in Y$$

Bridging Lemma for Composition Theorem



KEStub returns contents of shared mailbox

$X \subseteq Y$

$1 \in Y$

One-time Pad Step

- Next, we must define the SMC simulator `SMCSim`, and connect
  - `SMCReal(KEIdeal)`/`Adv’`
  - `SMCIdeal`/`SMCSim(Adv’)`

  where the input guard must exclude port index `3`
- This is done using EasyCrypt’s random sampling tactic
  - uses an isomorphism on the uniform distribution on exponents involving the plain text to be communicated
- We then apply the above when `Adv’` = `KESim(Adv)`

Overall Security Theorem

- Combining the instance of the composition theorem with the one-time pad step yields the connection between
  - `SMCReal(KEReal)`/`Adv`
  - `SMCIdeal`/`SMCSim(KESim(Adv))`

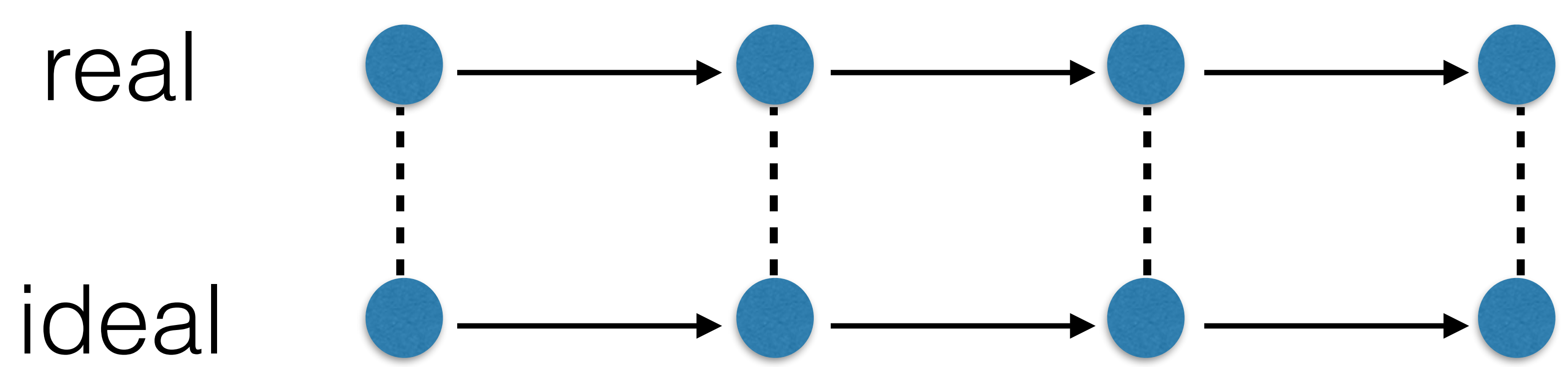
  where:
    - the input guard excludes `2` (from `KESim`) *and* `3` (from `SMCSim`)
    - the security upper bound is the DDH one, where the DDH adversary is applied to the composed environment

Lessons Learned

- SMC case study is complete, and validates our architecture and approach
- But it was too much work to scale-up to more realistic systems without some improvements to EasyCrypt and supporting tools

Relational Invariants/Symbolic Evaluation

- Proofs use *relational invariants* allowing the related evolutions of real and ideal games to be tracked
- Since the real and ideal worlds are structurally dissimilar, this means doing a lot of *symbolic evaluation*, essentially running code via tactics
- We have proposed and are implementing a way of automating this



Realization of UC Composition Theorem

- In our case study, we proved an instance of the UC Composition Theorem, via the definition of the composed environment and bridging lemmas
- We are now generalizing this work, producing a generic version of these definitions/proofs
- To obtain needed instances of the composition theorem, we'll then instantiate the generic definitions/proofs, and automatically generate some additional bridging definitions and proofs

Update from Paper

Dummy Adversary Lemma

- The same relational state may hold in two situations when the adversary is called:
  - when the adversary was called after the state was *first established*; or
  - when the adversary was invoked by the environment at stage when the state *already held*
- See the paper for how we currently unify these two cases in our proofs
- But we are working toward an improvement in which the user can think they are working in the so-called dummy adversary model — i.e., with an adversary that acts as instructed by the environment

Expressing Functionalities

- Defining real and ideal functionalities and simulators involves low-level message-routing code
- This boilerplate can be automatically generated, given domain specific language (DSL) for expressing functionalities and simulators
- DSL will be allow crypto theorists to more easily write and understand functionalities and simulators
- DSL type-checking will catch errors like badly formed messages, e.g., ones with bad source addresses
- Short term: translate DSL into existing EasyCrypt
- Longer term: integrate it into EasyCrypt

Conclusions

- The successful completion of our case study shows the validity of our UC in EasyCrypt architecture and approach
- But extensions and improvements to EasyCrypt and supporting tools will be needed for the approach to scale-up to realistic systems
- The EasyCrypt code for our case study, and a link to the extended (ePrint) version of our paper are available on GitHub:

`github.com/easyuc/EasyUC`