Solutions to assignment 4

1) By the division algorithm

$$q(x) = \frac{1}{2}x^2 - \frac{1}{4} \textit{ and } r(x) = -7x + \frac{5}{4}.$$

2) By the division algorithm

$$q(x) = 6x^2 + 3x + 5 \textit{ and } r(x) = 5x + 2$$

3) $4x^2 + 4x + 8$ (Euclidean algorithm).

4) $x + a + b$ (Euclidean algorithm).

5) Observe that constants $1, 2, 3, 4$ are the only invertible elements in $\mathbb{Z}_5[x]$, therefore the following are the only associates of $x^2 + x$:

$$x^2 + x, 2x^2 + 2x, 3x^2 + 3x, 4x^2 + 4x.$$

6) $x^3 + x + 1$, $x^3 + x^2 + 1$ (all others have roots in $\mathbb{Z}_2$).

7) It is not, $x^2 + x - 2 = (x - 1)(x + 2)$.

8) Observe that $a = a^5 (mod\ 5)$, so $x^5 + a = x^5 + a^5 = (x + a)^5$ in $\mathbb{Z}_5[x]$.