Math 235 01: Basic Algebra
Fall 2002

MIDTERM EXAM

Part I. Solve all the problems (10 points each):

1) Find $gcd(2613, 2171)$;

2) Find the remainder when 3^{2002} is divided by 5;

3) Prove that if $5|(a^2 + b^2 + c^2)$ then $5|a$, or $5|b$, or $5|c$;

4) Prove that $3\sqrt[3]{7} - 1$ is not a rational number;

5) Is it true that $(A \cup B) - (C \cup B)$ is equal to
   a) $A - C$? b) $(A - C) - B$? c) $A - (C \cap B)$?
   (show all the work).

Part II. Solve at least four problems (12 points each):

6) Prove that for any positive integer n $gcd(n + 1, n^2 - n + 1)$ is 1 or 3;

7) Prove that for any integer $n \geq 2$

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} \;>\; \sqrt{n}$$

8) Define a new addition $\oplus$ and multiplication $\odot$ on the set of integers Z by

$$a \oplus b = a + b - 1, \quad a \odot b = ab - (a + b) + 2.$$

   Prove that Z with these new operations is a commutative ring with 1 and without zero divisors;

9) Solve the equation $24x = 16$ in Z_{86};

10) For reals $a, b \in R$ denote

$$[a, b] = \{x \mid a \leq x \leq b\}, \quad [a, \infty) = \{x \mid a \leq x\}$$

   a) Is the function $x^2 + 2x - 6$ a bijection from $[0, \infty)$ to $[-10, \infty)$?
   b) Find a bijection between $[2, 4]$ and $[0, 10]$;
   c) Find a bijection between $[0, 1)$ and $[0, \infty)$ (Hint: use $tan(x)$).

Bonus Problem (10 points):

Let p, q be primes with $p \geq 5$ and $q \geq 5$. Prove that $24|(p^2 - q^2)$;

October 16th, 6:15-8:15pm, Adams Aud., Frank Dawson Adams Building, 3540 University.