Assignment 6

- 1) Determine whether the set G is a group under operation *:
 - a) $G = \{2^x \mid x \in \mathbb{Q}\}, a * b = ab;$
 - b) $G = \{x \in \mathbb{R} \mid x \neq -1\}, a * b = ab + a + b.$
 - c) $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \mid a, b \in \mathbb{R}, not \ both \ 0 \right\}$ with matrix multiplication *.
- 2) Let U_n be the group of units of the ring \mathbb{Z}_n .

a) List all elements of the group U_{10} and find their orders;

b) List all elements of order 4 in D_4 .

c) Write out the operation tables for U_6 , D_3 and the group $G = \{1, -1, i, -i\}$ of complex numbers with respect to multiplication.

3) a) List all elements of order 6 in $S_3 \times \mathbb{Z}_4$,

b) Give examples of non-abelian groups of orders 12, 16, 30, and 48.

- 4) If $(ab)^2 = a^2b^2$ for all a, b in a group G, prove that G is abelian.
- 5) Let $a, b \in G$ and ab = ba. If the orders |a| and |b| are relatively prime, prove that ab has order |a||b|.
- 6) a) Find the center of S_3 .
 - b) Is the subgroup H generated by $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ normal?
 - c) Write down all distinct cosets of the subgroup H from b).
- 7) Is the additive group $G = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ cyclic?
- 8) a) Are the groups $\mathbb{Z}_4 \times \mathbb{Z}_2$ and D_4 isomorphic?
 - b) Show that D_3 is isomorphic to S_3 .