MATH 235 01: Basic Algebra   Fall 2002

Assignment 4

1) Find polynomials $q(x)$ and $r(x)$ in $\mathbb{Q}[x]$ such that $f(x) = g(x)q(x) + r(x)$, where

$$f(x) = x^4 - 7x + 1, \quad g(x) = 2x^2 + 1 \ \ in\ \mathbb{Q}[x]$$

2) Find polynomials $q(x)$ and $r(x)$ in $\mathbb{Z}_7[x]$ such that $f(x) = g(x)q(x) + r(x)$, where

$$f(x) = 4x^4 + 2x^3 + 6x^2 + 4x + 5, \quad g(x) = 3x^2 + 2 \ \ in\ \mathbb{Z}_7[x]$$

3) Use the Euclidean Algorithm to find the gcd of the given polynomials:

$$x^5 + x^4 + 2x^3 - x^2 - x - 2 \ and\ x^4 + 2x^3 + 5x^2 + 4x + 4 \ \ in\ \mathbb{Q}[x]$$

4) Find the gcd of $x + a + b$ and $x^3 - 3xab + a^3 + b^3$ in $\mathbb{Q}[x]$.

5) List all associates of $x^2 + x$ in $\mathbb{Z}_5[x]$.

6) Find all irreducible polynomials of degree 3 in $\mathbb{Z}_2[x]$.

7) Is the polynomial $x^2 + x - 2$ irreducible in $\mathbb{Z}_3[x]$?

8) Show that $x^5 + a$ is reducible in $\mathbb{Z}_5[x]$ for each $a \in \mathbb{Z}_5$.