MATH 235 01: Basic Algebra Fall 2002

Assignment 2

1) Prove that $gcd(n, n+1) = 1$ for every integer n.

2) Find the smallest positive integer in the set $\{6u + 15v \mid u, v \in \mathbf{Z}\}$.

3) Prove that if a is odd then $4 \mid (a^2 - 1)$.

4) Prove that if a is odd and b is even then $gcd(a, b) = gcd(a + b, a - b)$.

5) Prove that if $3 \mid (a^2 + b^2)$ then $3 \mid a$ and $3 \mid b$.

6) Prove that $\sqrt{30}$ is irrational (i.e., $\sqrt{30}$ cannot be presented in the form $\frac{m}{n}$ where $m, n \in \mathbf{Z}$).

7) Show that $2^{11} - 1$ is not a prime number.

8) Find all positive integers n such that $n, n+2$, and $n+4$ are all prime.