



## GROUPS, LOGIC, AND COMPUTATION. GAGTA-2025

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*The homology of groupoids and groups associated to self-similar groups.*

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*Abstract:*

In the mid 2000s, Nekrashevych associated to each self-similar group a  $C^*$ -algebra, a group (called a Rover-Nekrashevych group) with simple commutator subgroup and a topological groupoid. Both the group and the  $C^*$ -algebra are easily constructible from the groupoid. Using Rubin's theorem, Nekrashevych showed that the groupoid can be recovered from the group. There are natural algebraic invariants associated to each of these objects. One has the K-theory of the  $C^*$ -algebra, the homology of the group and the homology of the groupoid. Xin Li showed that the rational homology of the groupoid determines the rational homology of the Rover-Nekrashevych group. If the groupoid is torsion-free, the homology determines the K-theory of the  $C^*$ -algebra through a spectral sequence.

In this talk I discuss recent work with Alistair Miller where we construct an exact sequence that connects the homology of the Nekrashevych groupoid to that of the self-similar group via the virtual endomorphism and transfer maps. There is a similar exact sequence for the K-theory of the Nekrashevych  $C^*$ -algebra. Using this exact sequence, we are able to explicitly compute the homology of the groupoids associated to the Grigorchuk group, the Grigorchuk-Erschler group, Gupta-Sidki  $p$ -groups and many other famous examples of self-similar groups. It follows from our results and Li's results that the Rover-Nekrashevych groups associated to the aforementioned self-similar groups are rationally acyclic.