Groups, Logic, and Computation. GAGTA-2025

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Undecidable Diophantine problems in groups and rings.

June 6–13, 2025

Abstract:
The Diophantine problem $\mathcal{D}(\mathcal{A})$ over an algebraic structure $\mathcal{A}$ is an algorithmic problem that asks to find an algorithm that, given a finite system of equations (with constants) over $\mathcal{A}$ outputs whether this system of equations has a solution in $\mathcal{A}$. If no such algorithm exists for a given $\mathcal{A}$, we say that $\mathcal{D}(\mathcal{A})$ is undecidable. Some of the classical examples include Hilbert's 10th problem stating that $\mathcal{D}(\mathbb{Z})$ (as a ring) is undecidable, and Makanin-Razborov algorithm establishing decidability of the Diophantine problem in free groups.

I will discuss recent results on undecidability of the Diophantine problem in nilptotent, solvable, and polycyclic groups. In the talk I will cover the reduction from $\mathcal{D}(G)$ for many classes of such groups to $\mathcal{D}(\mathcal{O})$ for some ring of algebraic integers $\mathcal{O}$. Combined with a recent result that $\mathcal{D}(\mathcal{O})$ is undecidable in all rings of algebraic integers, this establishes undecidability of the Diophantine problem in any such group. In particular, we prove that $\mathcal{D}(G)$ is undecidable in any non-virtually abelian nilpotent group, any metabelian group containing a non-virtually abelian nilpotent subgroup, etc. I will also cover undecidability of Diophantine problem in some classes of rings.

The talk is based on the joint work with Albert Garreta and Alexei Miasnikov.