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Classifying group actions on hyperbolic spaces.

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Abstract:

For a given group G, it is natural to ask whether one can classify all isometric G-actions on Gromov hyperbolic spaces. I will discuss a formalization of this problem based on the complexity theory of Borel equivalence relations. Our focus will be on actions of general type, i.e., non-elementary actions without fixed points at infinity, as these are particularly useful from the perspective of geometric group theory. The main result in this direction is the following dichotomy: for every countable group G, either all general type G-actions on hyperbolic spaces can be classified by an explicit invariant ranging in an infinitely dimensional projective space, or they are unclassifiable in a very strong sense. In terms of Borel complexity theory, we show that the equivalence relation associated with the classification problem is either smooth or K_σ-complete. Groups $SL_2(F)$, where F is a countable field of characteristic 0, satisfy the former alternative, while non-elementary hyperbolic (and, more generally, acylindrically hyperbolic) groups satisfy the latter. The proof of the main theorem draws on several results of independent interest and provides new insights into the boundary dynamics of group actions on hyperbolic spaces.

The talk is based on a joint paper with K. Oyakawa.