



GROUPS, LOGIC, AND COMPUTATION. GAGTA-2025

Markus Lohrey

University of Siegen

Streaming algorithms for (semi)groups.

June 6–13, 2025

Abstract:

The talk will be about highly efficient algorithms for word problems and more general membership problems in groups and semigroups: these algorithms should read the input word once from left to right symbol by symbol (such algorithms are known as streaming algorithms), spending ideally only constant time for each input letter. Moreover, the space used by the algorithm should be small, e.g. $O(\log n)$ or even $O(\log \log n)$ if n is the length of the input word. To achieve these goals we need in general randomization: the algorithm is allowed to make random guesses and at the end it should give a correct answer with high probability.

For word problems we obtain randomized streaming algorithms with the above requirements, using space $O(\log \log n)$ for nilpotent groups and commutative semigroups. The class of groups, for which the word problem can be solved by a randomized streaming algorithm in space $O(\log n)$ is quite reach: it contains all groups that can be obtained from finitely generated linear groups using finite extensions, graph products, and wreath products where the left factor is finitely generated abelian. On the other hand, using techniques from communication complexity, we can show that for some interesting groups (e.g. Thompson's group F) every randomized streaming algorithm for the word problem has space complexity $\Omega(n)$.

If time allows, I will also sketch a $O(\log n)$ -space streaming algorithm for the generalized word problem in a free group.