



## GROUPS, LOGIC, AND COMPUTATION. GAGTA-2025

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*Semigroups, groups, algebras, and their finitely presented expansions.*

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*Abstract:*

This is a joint work with Alexei Miasnikov. Finitely presented algebraic systems, e.g., groups, are fundamental in algebra and computation. They necessarily have computably enumerable word equality problems and are finitely generated (f.g.). Let us call such algebraic systems computably enumerable. This talk explores finitely presented expansions of computably enumerable algebraic systems. The expansion of algebraic systems, such as transforming groups into rings or introducing automorphisms, is an important technique in algebra, model theory, and theoretical computer science. Bergstra and Tucker proved that all computably enumerable algebraic systems with decidable word problems have finitely presented expansions. They, along with Goncharov, independently asked if every f.g. computably enumerable algebraic system has a finitely presented expansion. In this talk, we provide examples of f.g. computably enumerable semigroups, groups, and algebras that lack finitely presented expansions, thus answering the question of Bergstra-Tucker and Goncharov. These constructions rely on the interplay between key concepts from computability (e.g., immune sets), algebra (e.g., residual finiteness), and the theory of groups (e.g., Golod-Shafarevich theorem).