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On the Cannon-Thurston map for mapping tori of free group endomorphisms.

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Abstract:
Mitra proved that for any graph of groups satisfying the assumptions of the Bestvina-Feighn Combination Theorem and thus defining a word-hyperbolic group G, the inclusion of every vertex group H into G extends to the so-called Cannon-Thurston map between their hyperbolic boundaries, $\hat{i}\partial H \to \partial G$. In the case where H is normal in G, Mitra also provided a description of the fibers of $\hat{i}$ in terms of the so-called "ending laminations" associated with the boundary points of the quotient group G/H. If, in addition, H is either free or a closed hyperbolic surface subgroup, there are known results ensuring that, under suitable assumptions, $\hat{I}$ is uniformly finite-to-one. However, until now nothing has been known about the structure and properties of $\hat{i}$ if $H \leq G$ is not normal. We consider the groups $G_\phi =< F_r, t|txt^{-1} = \phi(x), x \in F_r >$, where F_r is free of finite rank $r \geq 2$ and where $\phi : F_r \to F_r$ is a fully irreducible injective non-surjective endomorphism of F_r with $\phi(F_r) \leq F_r$ malnormal. Then G_ϕ is word-hyperbolic and the Cannon-Thurston map $\hat{i}\partial F_r \to \partial G_\phi$ exists. We construct an "ending lamination" $\Lambda_\phi \subset \partial^2 F_r$ and prove that for two distinct points $p, q \in \partial F_r$ one has $\hat{I}(p) = \hat{i}(q)$ if and only if $(p, q) \in \Lambda_\phi$. We use this result to show that the map $\hat{i}\partial F_r \to \partial G_\phi$ is at most $2r$-to-one.

This talk is based on joint work in progress with Elizabeth Field.