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The first-order theory of torsion-free Tarski monsters.

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Abstract:

There has been various work studying the first-order theory of groups. For instance, Merzlyakov showed that the positive theories of all non-abelian free groups are the same. Vastly extending this is the solution of Tarski's problem by Miasnikov and Kharlampovich, and independently Sela, which says that the first-order theories of all non-abelian free groups are the same.

In this talk, we will discuss a method for controlling the first-order theory of certain small cancellation quotients of a torsion-free hyperbolic group. As an application, we construct a torsion-free Tarski monster (a group all of whose proper subgroups are infinite cyclic or trivial) that has trivial positive theory (namely, the same positive theory as a free group), answering questions of Casals-Ruiz, Garreta, and de la Nuez Gonzalez. Furthermore, the Tarski monster we construct has unbounded w-length but vanishing stable w-length for every non-trivial, non-surjective word w; and it does not admit an action on a hyperbolic space with a loxodromic element.

This is joint work with Rémi Coulon and Francesco Fournier-Facio.