GROUPS, LOGIC, AND COMPUTATION. GAGTA-2025

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How to blow up almost all words of a multiple context free language.

June 6–13, 2025

Abstract:
Originally introduced in 1987 in the search for a class of languages capturing the characteristics of human language, the class of multiple context free (mcf) languages has found its uses in the classification of groups according to the nature of their word problem. Muller and Schupp's theorem, which states that a finitely generated group has a context-free word problem if and onIy if it is virtually free, is an example, as context free languages belong the the class of mcf languages. Moreover, from work of Salvati, Ho, Gebhardt and Meunier it's now known that for all positive integers n, the groups $\mathbb{Z}^n$ have word problems which are mcf languages. In the other direction, Gilman, Kropholler and Schleimer proved that nilpotent groups and several right-angled Artin groups do not have mcf word problems. However the methods used to prove these (positive or negative) results do not extend to all right-angled Artin groups. In particular, at the time of writing it is not known whether or not the word problem of $F_2 \times \mathbb{Z}$ is an mcf language.

To prove a language is not regular or not context free it's common to employ a pumping lemma which in the context free case says something like: "any sufficiently long word w in the language can be factored as $w = uxvyw$, where uvw is short, xy is non empty and ux^ivy^iw is in the language, for all $i \geq 0$."

For mcf languages, outside the class of context free languages, this breaks down. The corresponding theorem, proved in [1] is much weaker, in that it applies only to a single word of the language. Worse, it has been shown that a natural generalisation of the context free pumping lemma to a (non context free) mcf language cannot apply to the complement of any finite subset of the language.

In this talk, I will define the class of mcf languages and describe an alternative to the pumping lemma which, although not as symmetric as the classic pumping lemma, does apply to almost all words of a mcf language. I will then explain how this can be used to show a language is not mcf.

Joint work with Murray Elder, Lisa Frenkel and Mengfan Lyu.