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n-Engel groups for large n.

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Abstract:

n-Engel group is a group that satisfies a group law $[x, y, \dots y] = 1$, where y is repeated n times. The following natural question was asked in the 1950s: if such a group is locally nilpotent. For $n \leq 4$ the answer is affirmative. For lager n this question seems to be quite delicate. Namely, if one adds some extra restrictions together with the n-Engel identity, the resulting group is indeed locally nilpotent. For example, finitely generated n-Engel groups that are at the same time residually finite, or solvable are indeed nilpotent. However, we expect that finitely generated free n-Engel groups are not nilpotent for sufficiently large n.

The Engel problem has a connection with the Burnside problem, which asks whether a finitely generated group with a group law $x^n = 1$ is necessarily finite. The new proof of the Burnside problem was recently obtained by E. Rips, K. Tent and myself. The ideas from this proof help to make progress in the Engel problem. I will talk about the current progress and difficulties along the way.

This is a joint work with K. Tent.