Manhattan Algebra Day

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An extension of the O'Nan-Scott Theorem

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Abstract:
Any totally disconnected locally compact group G is known by van Dantzig's Theorem to contain a totally disconnected compact open subgroup U. If Ω denotes the set of right cosets of U in G, then G acts on Ω, and the subdegrees (orbits of point-stabilizers) of G under this action are all finite. In this way every totally disconnected locally compact group admits a permutation representation that is *subdegree finite*. Little is known in general about the structure of these permutation groups.

It is usual when examining permutation groups to look first at the *primitive* permutation groups (transitive groups in which point stabilizers are maximal); in the finite case these groups are the basic building blocks from which all finite permutation groups are comprised. Thanks to the seminal O'Nan–Scott Theorem and the Classification of the Finite Simple Groups, the structure of finite primitive permutation groups (which are necessarily subdegree finite) is broadly known. For infinite, subdegree finite, primitive permutation groups, very little is known.

In this talk I'll describe a new theorem of mine which extends the O'Nan–Scott Theorem to a classification of all primitive permutation groups with finite point stabilizers. This theorem describes the structure of these groups in terms of finitely generated simple groups.

One of the many nice consequences of this theorem is a description of those totally disconnected locally compact groups G which contain a compact open subgroup U (whose existence is guaranteed) which satisfies:

1. U is a maximal subgroup of G; and
2. some finite-index subgroup of U is normal in G.