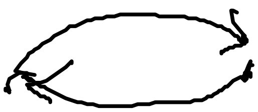
1) Cayley complexes

$$G = \langle X \mid R \rangle, \quad X = X^{-1}$$

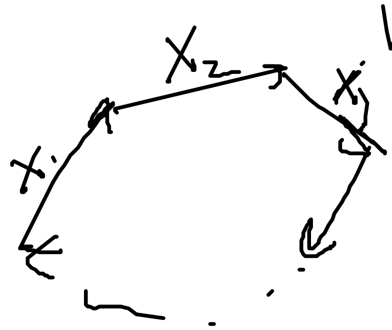
$Cay_X(G)$

$V = G$ $\qquad$ E: $g \xrightarrow{x} gx$ $\quad x \in X$

$\qquad\qquad\qquad\quad gx \xrightarrow{x^{-1}} g$



. Pick origin, e . $w =_G 1 \iff w$ is a loop in C.



$W = 1$
G

$G = \langle X \mid R \rangle$

$r \in R$

Length space $C_G(X)$

Simply connected

$D^2 \to C_G ; \partial(D^2) \to$

loop in $C_X(G)$: ℓ

$lab(\ell) =_G 1$

$g : \partial(D^2) \to CC$

can be extended to a map

$D^2 \to CC$

$\Rightarrow$ CC is s. c.

[5] I.1 R is finite $\Rightarrow$ CC is l.c.

---

1 b

$\cancel{g}\, hg' : g \longrightarrow h$

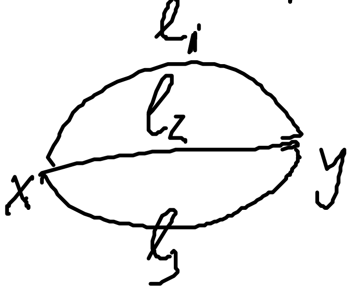
in CC – coarsely hom.

C – hom.

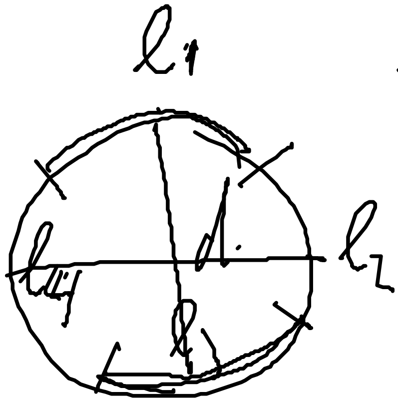
Areas of metric spaces

1) Bowditch

A: loop $\to \mathbb{R}_+$

a) 

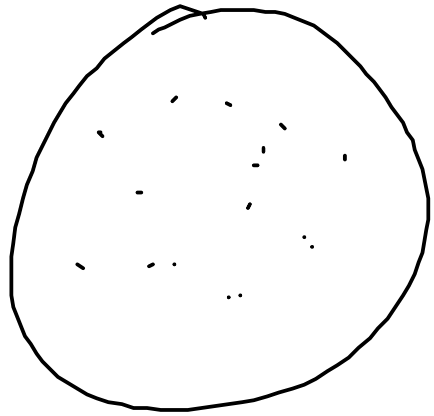
$$A(l_1 l_2^{-1}) + A(l_2 l_3^{-1}) \geq$$
$$\geq A(l_1 l_3^{-1})$$

b) 

$$A(l) \geq K d_1 d_2$$

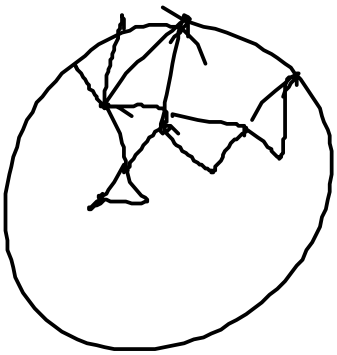
2) <u>Gromov</u>

A_δ



ℓ bounds a disc

$\varphi: D^2 \longrightarrow X \qquad \partial(D^2) \to \ell$

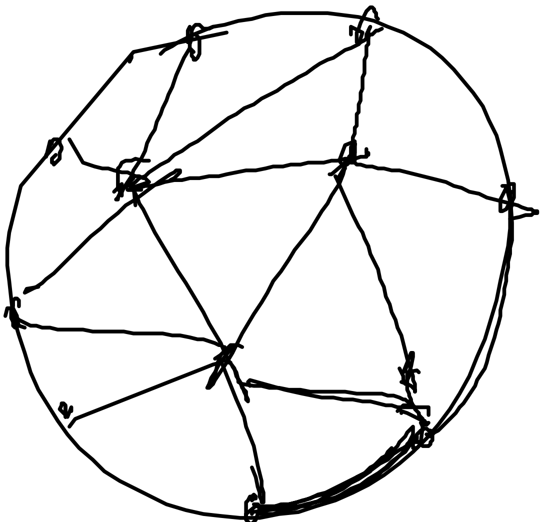


<u>and φ continuous</u>

$\exists\, \varepsilon,\ \exists$ <u>net</u> $N \subseteq X$

$\mathrm{dist}(\varphi(n_1), \varphi(n_2)) \leq \delta$

$\mathrm{dist}(n_1, n_2) < \varepsilon$.



Area = number of triangles

Lipschitz maps

$\mathrm{dist}(\varphi(a), \varphi(b)) \leq \alpha \, \mathrm{dist}(a, b)$

Bi-L map

$C_1 > 1$

$$\frac{1}{C_1} d(a,b) < d(\varphi(a), \varphi(b)) < C_1 d(a,b)$$

Quasi-isometry

$$\frac{1}{C_1} d(a,b) - C_2 < d(\varphi(a), \varphi(b)) < C_1 d(a,b) + C_2$$

$$\mathbb{R} \underset{q.i.}{\cong} \mathbb{Z}$$

$$X \underset{q.i.}{\simeq} Y$$

$$\exists X' \subseteq X, \quad Y' \subseteq Y$$

$$\text{s} \quad \varphi : X' \underset{q.i.}{\longrightarrow} Y'$$

$$\mathrm{dist}_H(X, X') < \infty \quad \mathrm{dist}_H(Y, Y') < \infty$$

$$\mathbb{Z}^2 \cong \mathbb{R}^2$$

Asymptotic cones

X - metric space, d

$$X/p = (X, d/p)$$

Take a sequence of sc. constants

$$p_1 < p_2 < \dots < \to \infty$$

$$X_i = X/p_i$$

$\mathbb{Z}$ . . . .

$\mathbb{Z}/2$ . . . .

$$\varinjlim X_i = \mathrm{Con}(X)$$

$$\lim X_i = \{(x_i) \mid x_i \in X_i\}$$

$$\mathrm{dist}\big((x_i),(y_i)\big) = \varinjlim d(x_i, y_i)/p_i$$

ω – ultrafilter – set of big sets

a) $\emptyset$ is small

b) complement of big is small

c) A big, $B \supseteq A \Rightarrow B$ is big

$\exists\, \omega \Leftarrow \underline{AC}$

<u>or</u> ω is a f. additive measure on $P(\mathbb{N})$: $\omega(\emptyset) = 0$

ω – a.s. (ω is a measure).

$\lim^{\omega} a_i = b$ if $\forall \varepsilon$

ω – a.s. $|a_i - b| < \varepsilon$

If we pick ω. every sequence has a limit.

$1, 2, 1, 2, 1, 2, \ldots \longrightarrow$ 1 if $2\mathbb{N}+1 \in \omega$, 2 if $2\mathbb{N} \in \omega$

<u>Th</u> $\forall a_i \ \exists \lim^{\omega} a_i$

a) if unbounded $\Rightarrow \pm\infty$

b) $a_i \in [p, q]$ ω – a.s.

ω – a.s. $a_i \in$ first half

continue, $\bigcap [p_i, q_i] = \{b\}$

$Con^{\omega}(X, (p_i)) = \{(x_i) \mid x_i \in X\}$

$$\operatorname{dist}((x_i),(y_i)) = \lim^{\omega} \frac{d(x_i, y_i)}{p_i}$$

a) $\operatorname{dist} = \begin{cases} +\infty \\ < \infty, \neq 0 \\ 0 \end{cases}$

$+\infty$: pick a point (O_i)

$$\operatorname{Con}^{\omega}(X_i, (p_i)) = \{ (x_i) \mid \operatorname{dist}((x_i),(O_i)) < \infty \}$$

$$\frac{d(x_i, O_i)}{d_i} < k \quad \Big| \quad d(x_i, O_i) < k d_i = O(d_i)$$

$\sim$ $\quad (x_i) \sim (y_i)$ if $d((x_i),(y_i)) = 0$

$$\operatorname{Con}^{\omega}(X_i, (p_i), (O)) = \{ (x_i) \mid \operatorname{dist}((x_i),(O_i)) < \infty \} / \sim$$