Elementary Theory of Finitely Generated Nilpotent Groups

Mahmood Sohrabi, Mcgill University
(joint with Alexei G. Myasnikov, McGill University)

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ACC Webinar

Outline

- Groups (not necessarily finitely generated) elementarily equivalent to a free nilpotent group of finite rank.
- Limits of free nilpotent groups
- Elementary equivalence and quasi-isometry of nilpotent group

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Elementary theories

Definition

The elementary theory $Th(\mathcal{A})$ of a group $\mathcal{A}$ (or a ring, or an arbitrary structure) in a language L is the set of all first-order sentences in L that are true in $\mathcal{A}$.

Definition

Let $\mathcal{L}$ be a language. A formula Φ is a **universal formula** if $\Phi = \forall x_1, \ldots, \forall x_n \Psi$ and Ψ is quantifier free. The set of all universal sentences $Th_{\forall}(G)$ true in G is called the **universal theory** of G.

Definition

Two groups (rings) $\mathcal{A}$ and $\mathcal{B}$ are elementarily equivalent in a language L ($\mathcal{A} \equiv \mathcal{B}$) if $Th(\mathcal{A}) = Th(\mathcal{B})$ (i.e., they are undistinguishable in the first order logic in L). the structures $\mathcal{A}$ and $\mathcal{B}$ are universally equivalent if $Th_{\forall}(\mathcal{A}) = Th_{\forall}(\mathcal{B})$.

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Tarski type questions

Classification of groups and rings by their first-order properties goes back to A. Tarski and A. Mal'cev.

Questions concerning universal theories have become important much more recently and have interesting connections with many other concepts.

Elementary classification problem

Characterize in algebraic terms all groups (rings), perhaps from a given class, elementarily equivalent to a given one.

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Tarski

Complex numbers $\mathbb{C}$

$Th(\mathbb{C}) = Th(F)$ iff F is a char 0 algebraically closed field.
$Th_\forall(\mathbb{C}) =$ The theory of char 0 integral domains.

For a given abelian group A one can define a countable set of elementary invariants $I(A)$ (which are natural numbers or ∞).

Theorem [W.Szmielew]

Let A and B be abelian groups. Then:

$$A \equiv B \Leftrightarrow I(A) = I(B).$$

If $\mathbb{Z}$ denotes the infinite cyclic group then $Th_\forall(\mathbb{Z}) = Th_\forall(\mathbb{Z}^n)$ for any n. Though $Th(\mathbb{Z}) \neq Th(\mathbb{Z}^n)$.

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Nilpotent groups

Definition

A group G is called **nilpotent** if the lower central series:

$$\Gamma_1(G) =_{df} G, \quad \Gamma_{i+1}(G) =_{df} [G, \Gamma_i(G)], i \geq 1,$$

is eventually trivial.

Definition

A group G is called **free nilpotent of rank** r **and and class** c if

$$G \cong F(r)/\Gamma_{c+1}(F(r)),$$

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Groups elementarily equivalent to a divisible nilpotent group

Definition

If G is torsion free f.g nilpotent group and R is binomial domain then by G^R is meant the P.Hall R-completion of G.

Theorem (Myasnikov, 1984)

Suppose that G is a directly indecomposable torsion free f.g. nilpotent group and H is a group. Then

$$H \equiv G^{\mathbb{Q}} \Leftrightarrow H \cong G^F,$$

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Theorem (A.Myasnikov, 1984)

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Suppose that G and H are f.g. nilpotent groups. Then

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Theorem (O. Belegradek, 1992)

For a group H one has

$$H \equiv UT_n(\mathbb{Z}) \Leftrightarrow H \text{ is "quasi"} - UT_n(R),$$

some $R \equiv \mathbb{Z}$.

Statement of the main Result

Notation

$$N_{r,c}(\mathbb{Z}) = F(r)/\Gamma_{c+1}(F(r))$$

$$N_{r,c}(R) = \text{ Hall } R\text{-completion of } N_{r,c}(\mathbb{Z}).$$

Theorem (A. Myasnikov, MS)

Assume that G is a group. Then

$$G \equiv N_{r,c}(\mathbb{Z}) \Leftrightarrow G \textit{ is a quasi-}N_{r,c} \textit{ group over } R,$$

for some $R \equiv \mathbb{Z}$. *Moreover there exists* $G \equiv N_{r,c}(\mathbb{Z})$ *where G is not* $N_{r,c}$.

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Lemma

Assume $\{g_1, \ldots, g_r\}$ is a free generating set for $G = N_{r,c}(R)$ as an R-group. Then

$$C_G(g_j) = g_j^R \oplus Z(G).$$

Rough definition of a quasi-$N_{r,c}$ group

Let $G = N_{r,c}(R)$ and $\{g_1, \ldots, g_r\}$ be a generating set for G. Then a **quasi**-$N_{r,c}$ group H over R has the following description

- $G = H$ as sets
- every aspect of H is the same as G except $C_H(g_j)$ is an abelian extension of $Z(H)$ by $C_H(g_j)/Z(H) \cong R^+$.

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A proof sketch

Lemma

Assume $G = N_{r,c}(R)$ then R is interpretable in G and the action of R on all the quotients Γ_i/Γ_{i+1} is absolutely interpretable in G.

Now use the fact that multiplication in G is defined using certain polynomials (which can be viewed as terms of the language of rings) to pull back the interpretability of the action of R on the quotients to the group.

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Lemma

Let $G = N_{r,c}(R)$.

- The action of R on $[G, G]$ is interpretable in G.
- If $\{g_1, \ldots, g_r\}$ is any free generating set for G as an R-group, then the abelian subgroups

$$g_j^R \cdot Z(G)$$

$1 \leq j \leq r$ are definable in G. However g_j^R are not so!

Lemma

If $H \equiv N_{r,c}(R)$ Then there exists a ring $S \equiv R$ such that the formulas that interpret the rings R and S and their respective actions on $\Gamma_i(G)/\Gamma_{i+1}(G)$ and $\Gamma_i(H)/\Gamma_{i+1}(H)$ are the same.

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Thus the information in the previous lemma carries over to H with R replaced by S.

Definition

A **marked group** (G, S) is a group G together with an ordered generating set $S \in G^n$. The set S is called a marker for G.

Definition

The distance between two marked groups (G, S) and (G', S') is at most $e^{-(2R+1)}$ if $B_{Caley(G,S)}(1, R)$ and $B_{Caley(G',S')}(1, R)$ coincide when the markers are identified.

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Let $\mathcal{C}$ be a class of groups. A group G is called **fully residually** $\mathcal{C}$ if for any finite $X \subseteq G$ there is a homomorphism $\phi : G \to H \in \mathcal{C}$ injective on X.

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Theorem

Let G and H be groups. Then TFAE:

1. The marked group (G, S) is the limit of some markings (H, S_i) of H.
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3. $Th_\forall(H) \subseteq Th_\forall(G)$.
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Questions

What are the limits of free nilpotent groups? In particular what are the limits of the Heisenberg group $UT_3(\mathbb{Z})$?
Is it true that every limit of $UT_3(\mathbb{Z})$ embeds in $UT_3(\mathbb{Z}[x])$?

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- Every free 2-nilpotent group is the limit of some markings of $UT_3(\mathbb{Z})$.
- For any n the group $UT_3(\mathbb{Z}) \oplus \mathbb{Z}^n$ is the limit of some markings of $UT_3(Z)$.
- All of the examples above embed in $UT_3(\mathbb{Z}[x])$.

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Logic and large scale geometry

Proposition

Let $G \equiv G'$ be two f.g. nilpotent groups. Then they are quasi-isometric.

Proposition

If G is a f.g. group quasi-isometric with $UT_3(\mathbb{Z})$ then G is weakly commensurable with (an elementarily equivalent copy of) $UT_3(\mathbb{Z})$.

Questions

How the elementary theory and quasi-isometries of f.g. nilpotent groups relate? The proofs of the statements above are purely algebraic. Is there a more direct (logical) way of proving the statements? Will looking at Caley graphs as logical objects help? What is the right language and context for such a purpose?

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If G is a f.g. group quasi-isometric with $UT_3(\mathbb{Z})$ then G is weakly commensurable with (an elementarily equivalent copy of) $UT_3(\mathbb{Z})$.

Questions

How the elementary theory and quasi-isometries of f.g. nilpotent groups relate? The proofs of the statements above are purely algebraic. Is there a more direct (logical) way of proving the statements? Will looking at Caley graphs as logical objects help? What is the right language and context for such a purpose?