



ALGEBRA DAY ON THE HUDSON

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Classifying group actions on hyperbolic spaces.

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Abstract:

Studying groups via their actions on Gromov hyperbolic spaces has been a recurrent theme in geometric group theory over the past three decades. Of particular interest in this approach are actions of general type, i.e., non-elementary actions without fixed points at infinity. For a given group G , it is natural to ask whether it is possible to classify all general type actions of G on hyperbolic spaces. In a joint paper with D. Osin, we propose a formalization of this question based on the complexity theory of Borel equivalence relations. Our main result is the following dichotomy: for every countable group G , general type actions of G on hyperbolic spaces can either be classified by an explicit invariant ranging in the infinite-dimensional projective space or are unclassifiable by countable structures. Special linear groups over countable fields provide examples satisfying the former alternative, while every non-elementary hyperbolic group satisfies the latter.