

“Group Theory International” Online Seminar

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“Equations in free groups and EDTOL languages”

Thursday, April 16, noon (New York Time)

Let $F = F(A)$ be a free group and $W = 1$ be an equation with constants in A and variables X_1, \dots, X_m over F .

In a joint work with Laura Ciobanu and Murray Elder we showed that the set of all solutions of W is an EDTOL language. This means: there is a nondeterministic finite automaton (NFA) with the following properties.

1. The labels of the automaton are endomorphisms over a free monoid with involution C^* .
2. We have $(A \cup A^{-1})^* \subseteq C$.
3. There is a special symbol “#” in the alphabet C such that the set of all solutions in reduced words is exactly the set of all $h(\#)$, where h ranges through the rational set of endomorphisms R accepted by the NFA.

Moreover, the NFA has exponential size in the input specification for W .

As a consequence of the construction we obtain an improved complexity for deciding the existential theory for free groups. It is in $\text{NSPACE}(n \log n)$, which we believe to be space optimal. Within the same complexity we can decide whether or not the solution set is finite. We apply the compression technique due to Jež which provides the simplest method for deciding the existential theory. Our results generalize to free products of free and finite groups and cope with rational constraints.

Next presentation: Apr 30, TBA