Algorithmic problems in the groups of the form $F/N^{(d)}$

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$$M(X;N) = \left\{ \left(\begin{array}{cc} g & \pi \\ 0 & 1 \end{array} \right) \middle| \, g \in F/N, \ \pi \in \mathcal{F}_\Gamma \right\};$$

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  (b) conjugacy problem in $M(X;N)$ is decidable if and only if conjugacy and power problem are decidable in F/N.

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- Later Remeslennikov and Sokolov [1970] extended (a) to any torsion free group F/N and also showed that power problem is decidable in free solvable groups, and deduced that free solvable groups have decidable conjugacy problem.

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- Later Remeslennikov and Sokolov [1970] extended (a) to any torsion free group F/N and also showed that power problem is decidable in free solvable groups, and deduced that free solvable groups have decidable conjugacy problem.
- Finally, C. Gupta [1982] proved that (a) holds for groups with torsion and that for any group F/N:

$$\left\{ \begin{array}{l} \mathbf{CP}(F/N) \\ \mathbf{PP}(F/N) \end{array} \right. \Rightarrow \mathbf{CP}(F/N').$$

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  (b) $\mathbf{CP}(F/N)$ is decidable if and only if $\mathbf{CP}(F/N')$ is decidable.
  (c) $\mathbf{WP}(F/N')$ is decidable if and only if $\mathbf{CP}(F/N')$ is decidable.

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$\mathbf{CP}(F/N) \not\Rightarrow \mathbf{CP}(F/N')$.

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- **X-complete** if for every $v \in V$ and $x \in X$ there exists an edge e with $\mathbf{o}(e) = v$ and $\mu(e) = x$;
- **inverse** if with every edge $e = (g_1, g_2, x)$, Γ also contains the inverse edge $e^{-1} = (g_2, g_1, x^{-1})$.

Scherier Graph

Let $F = F(X)$ and $H \leq F$. The *Schreier graph* of the subgroup H, denoted by $\mathbf{Sch}(X; H)$, is an X-digraph (V, E), where V is the set of right cosets

$$V = \{Hg \mid g \in F\}$$

and

$$E = \{Hg \stackrel{x}{\to} Hgx \mid g \in F,\ x \in X^{\pm}\}.$$

Cayley Graph

$\mathbf{Sch}(X; H)$ is an inverse folded complete X-digraph with root H. A special case of the Schreier graph is when $H = N \trianglelefteq F$, called a Cayley graph of the group F/N denoted by $\mathbf{Cay}(X; N)$.

Flows on inverse X-digraphs

Let $\Gamma = (V, E)$ be an inverse X-digraph. A function $f : E \to \mathbb{Z}$ defines the function $\mathcal{N}_f : V \to \mathbb{Z}$:

$$\mathcal{N}_f(v) = \sum_{\mathbf{o}(e)=v} f(e),$$

called the **net-flow** function of f. We say that f is a **flow** if it satisfies the following conditions.

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(F3) There exist $s, t \in V$ such that $\mathcal{N}_f(v) = 0$ for all $v \in V \setminus \{s, t\}$, and $\mathcal{N}_f(s) = 1$ and $\mathcal{N}_f(t) = -1$. If f is called a flow from the **source** s to the **sink** t.

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A flow f is called a **circulation** if $\mathcal{N}_f(v) = 0$ for all $v \in V$.

Flows defined by words

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The word w defines a unique path p_w in Γ:

$$v_0 \xrightarrow{x_{i_1}^{\varepsilon_1}} v_1 \xrightarrow{x_{i_2}^{\varepsilon_2}} v_2 \xrightarrow{x_{i_3}^{\varepsilon_3}} \dots \xrightarrow{x_{i_k}^{\varepsilon_k}} v_k$$

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where v_0 is the root of Γ, and a function

$$\pi_w^\Gamma : E \to \mathbb{Z}$$

$$\pi_w^\Gamma(e) = \#\ of\ times\ e\ is\ traversed - \#\ of\ times\ e^{-1} is\ traversed\ by\ p_w.$$

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It can be easily checked that π_w^Γ is a flow in Γ. We call π_w^Γ the **flow** of w in Γ.

Word Problem: $\mathbf{WP}(F/N) \Rightarrow \mathbf{WP}(F/N')$

Lemma

Let $H \leq F$, $\Delta = \mathbf{Sch}(X;H)$, *and* $w \in F$. *Then* $\pi_w^\Delta = 0$ *if and only if* $w \in [H,H]$.

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Proposition

If the word problem is decidable in F/N, *then it is decidable in* F/N'.

Auslander-Lyndon:1955

Theorem

The operation of F/N on N/N' is effective; that is, only unit element of F/N leaves all elements of N/N' fixed. The operation of F/N is induced by the inner automorphisms of F.

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which is equivalent to:

$$\begin{aligned} v \in N &\Leftrightarrow v^{-1}w^{-1}v = w^{-1} \quad \forall w \in N/N' \\ &\Leftrightarrow v^{-1}w^{-1}vw = 1 \\ &\Leftrightarrow [v, w] = 1 \\ &\Leftrightarrow [v, w] \in [N, N], \quad \forall w \in N. \end{aligned}$$

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Assume that N is a recursively enumerable normal subgroup of F and N' is recursive, then N is recursive.

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Proof.

The statement is obvious for abelian F or $N = \{1\}$. Assume that F is not abelian and N is not trivial. Then N has rank at least 2. By Theorem A-L, for any $w \in F \setminus N$ there exists $r \in N$ such that $[w, r] \notin N'$. That gives a procedure for testing if $w \notin N$ making N recursive. □

Corollary

Assume that N is recursively enumerable normal subgroup of F and $\mathbf{WP}(F/N^{(d)})$ *is decidable for some* $d \in \mathbb{N}$*. Then* $\mathbf{WP}(F/N^{(d)})$ *is decidable for every* $d \in \mathbb{N}$*.*

Corollary

Assume that N is recursively enumerable normal subgroup of F and $\mathbf{WP}(F/N^{(d)})$ is decidable for some $d \in \mathbb{N}$. Then $\mathbf{WP}(F/N^{(d)})$ is decidable for every $d \in \mathbb{N}$.

$$\mathbf{WP}(F/N) \longleftrightarrow \mathbf{WP}(F/N') \longleftrightarrow \mathbf{WP}(F/N'') \longleftrightarrow \dots$$

F/N' is torsion free

Definition

The function $\|\cdot\| : \mathcal{F}_\Gamma \to \mathbb{Z}$ defined by:

$$\|\pi\| = \sum_{e \in E^+} |\pi(e)|$$

is called a *norm* on $\mathcal{F}_\Gamma$.

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Lemma

For every $w \notin N'$ and $k \in \mathbb{N}$ we have $\|\pi^{\Gamma}_{w^k}\| \geq k$.

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Proof.

By the previous lemma if $w \notin N'$ and $k \in \mathbb{N}$, then $\|\pi^{\Gamma}_{w^k}\| \geq k$, i.e., $w^k \notin N'$. □

Power Problem

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Proof.

If $|v| < k$, then $\|\pi_v\| < k \leq \|\pi_{u^k}\|$, which means that $u^k \neq v$ in F/N'. □

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If $\mathbf{WP}(F/N)$ *is decidable, then* $\mathbf{PP}(F/N')$ *is decidable.*

Proof.

By the previous lemma, given $u, v \in F$ it is sufficient to check if $v = u^k$ in F/N' for $k = -|v|, \ldots, |v|$ which reduces to $2|v| + 1$ number of times solving the word problem in F/N' for the words $v^{-1}u^{-|v|}, \ldots, v^{-1}u^{|v|}$ whose lengths are bounded by $|v| + |u| \cdot |v|$. □

Magnus Embedding

The set of matrices:

$$M(X;N) = \left\{ \left(\begin{array}{cc} g & \pi \\ 0 & 1 \end{array} \right) \middle| \, g \in F/N, \ \pi \in \mathcal{F}_\Gamma \right\}$$

forms a group with respect to the matrix multiplication and it can be also recognized as the wreath product $\mathbb{Z}^n \, \mathbf{wr} \, F/N$.

$(\mathcal{F}_\Gamma, +)$ as a f.g. free $\mathbb{Z}F/N$-module of rank n and Magnus Embedding

Lemma

Let π_{x_i} be denoted by π_i for $i = 1, \dots, n$, then $\mathcal{F}_\Gamma$ is a free $\mathbb{Z}F/N$-module of rank n with a free basis $\{\pi_1, \dots, \pi_n\}$. In particular, every $\pi \in \mathcal{F}_\Gamma$ can be uniquely expressed as a $\mathbb{Z}F/N$ linear combination of $\pi_1, \dots, \pi_n$.

Magnus Embedding

Let $^{-} : F \to F/N$ be the canonical epimorphism. Define a homomorphism $\varphi : F \to M(X; N)$ by:

$$x_i \overset{\varphi}{\mapsto} \begin{pmatrix} \overline{x}_i & \pi_i \\ 0 & 1 \end{pmatrix}, \quad x_i^{-1} \overset{\varphi}{\mapsto} \begin{pmatrix} \overline{x}_i^{-1} & -\overline{x}_i^{-1}\pi_i \\ 0 & 1 \end{pmatrix}. \qquad (1)$$

It is easy to check by induction on $|w|$ that:

$$\varphi(w) = \begin{pmatrix} \overline{w} & \pi_w \\ 0 & 1 \end{pmatrix}.$$

Magnus Embedding

Theorem (Magnus Embedding)

Let $F = F(x_1, \ldots, x_n)$, $N \trianglelefteq F$, *and* $^- : F \to F/N$ *be the canonical epimorphism. The homomorphism* $\varphi : F \to M(X; N)$ *defined by*

$$x \stackrel{\varphi}{\mapsto} \begin{pmatrix} \overline{x} & \pi_i \\ 0 & 1 \end{pmatrix}$$

satisfies $\ker(\varphi) = N'$. *Therefore,* $F/N' \simeq \varphi(F) \leq M(X; N)$. *The induced embedding* $\mu : F/N' \to M(X; N)$ *is called the* **Magnus embedding**. □

Conjugacy problem

Matthews proved that:

$$\mathbf{CP}(M(X;N)) \Leftrightarrow \begin{cases} \mathbf{CP}(F/N), \\ \mathbf{PP}(F/N). \end{cases}$$

Now what we have is that restricting the conjugacy problem from $M(X;N)$ to F/N' gives a problem equivalent to $\mathbf{PP}(F/N)$. In general, decidability of $\mathbf{CP}(F/N)$ is irrelevant to decidability of $\mathbf{CP}(F/N')$.

Magnus Embeding is Fratini

The theorem below was first proved by Remeslennikov and Sokolov for a torsion free group F/N and by C. Gupta for any finitely generated group F/N.

Theorem

For any $u, v \in F$ the matrices

$$\mu(u) = \begin{pmatrix} \overline{u} & \pi_u^{\Gamma} \\ 0 & 1 \end{pmatrix} \text{ and } \mu(v) = \begin{pmatrix} \overline{v} & \pi_v^{\Gamma} \\ 0 & 1 \end{pmatrix}$$

are conjugate in $M(X;N)$ if and only if they are conjugate in $\mu(F/N')$.

Theorem (Geometry of conjugacy problem)

Let $N \trianglelefteq F$, $u, v \in F$, and $\Delta = \mathbf{Sch}(X, \langle N, u\rangle)$. Then $u \sim v$ in F/N' if and only if there exists $c \in F$ satisfying the conditions:

(a) $\pi_u^\Delta = \overline{c}\pi_v^\Delta$, *i.e., π_u can be obtained by a $\overline{c}$-shift of π_v in Δ;*

(b) $\overline{c}^{-1}\overline{uc} = \overline{v}$ *in F/N.*

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Theorem

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Proof.

We may assume that $u \neq 1$ and $v \neq 1$ in F/N'. If $u = 1$ in F/N, then $\pi_u^\Delta = \pi_u^\Gamma \neq 0$. If $u \neq 1$ in F/N, then we get again that: $\pi_u^\Delta \neq 0$. It shows that Case (2) in the proof of Matthews is impossible in F/N' and allows us to drop decidability of $\mathbf{CP}(F/N)$. The rest of the proof is essentially the same as the proof of Matthews.

□

$\mathbf{CP}(F/N') \Rightarrow \mathbf{PP}(F/N)$

Proposition

Let $N \trianglelefteq F$ and $u, v \in F \setminus N$ satisfy $[u, v] = 1$ in F/N. Then $v \in \langle u \rangle$ in F/N if and only if $u \sim u[w, v]$ in F/N' for every $w \in \langle N, u \rangle$.

Theorem

Assume that N is recursively enumerable. Then $\mathbf{CP}(F/N') \Rightarrow \mathbf{PP}(F/N)$.

$\mathbf{CP}(F/N') \Rightarrow \mathbf{PP}(F/N)$

Proof.

By assumption $\mathbf{CP}(F/N')$ is decidable. Hence $\mathbf{WP}(F/N')$ is decidable and $\mathbf{WP}(F/N)$ is decidable. Consider an arbitrary instance $u, v \in F$ of $\mathbf{PP}(F/N)$. Our goal is to decide if $v \in \langle u \rangle$ in F/N, or not.

- If $v = 1$ in F/N, then the answer is YES.
- If $u = 1$ in F/N and $v \neq 1$, then the answer is NO.
- If $[u, v] \neq 1$ in F/N, then the answer is NO.

Hence, we may assume that $u \neq 1$, $v \neq 1$, and $[u, v] = 1$ in F/N. To test if $v \in \langle u \rangle$ in F/N we run a process that checks if $v = u^k$ in F/N for some $k \in \mathbb{Z}$. To test if $v \notin \langle u \rangle$ in F/N we enumerate all words $w \in \langle N, u \rangle$ and solve the conjugacy problem for words u and $u[w, v]$ in F/N'. By the previous proposition, if $v \notin \langle u \rangle$ then a negative instance will be found eventually. □

Theorem

There exists a recursive $N \trianglelefteq F$ *with undecidable* $\mathbf{CP}(F/N)$ *and decidable* $\mathbf{CP}(F/N')$.

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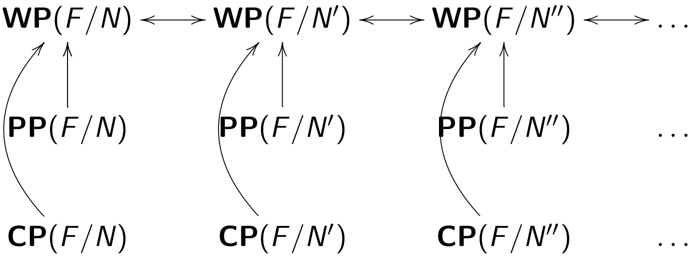
Proof.

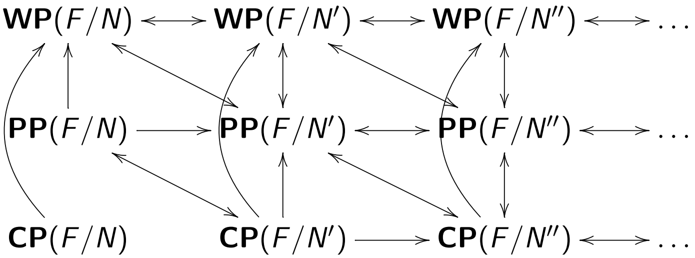
C. Miller constructed a group $G(U)$ from a group U with a finite presentation such that $G(U)$ has a decidable power problem. He also proved that $\mathbf{CP}(G(U))$ is decidable if and only if $\mathbf{WP}(G(U))$ is decidable. Thus choosing a finitely presented group U with undecidable word problem we obtain a group $G(U)$ with the required property. □

Relations among the algorithmic problems without our results



Summary: Relations among the algorithmic problems together with our results

Theorems we stated so far give the following diagram of problem reducibility for a finitely generated recursively presented group F/N:



THANK YOU