STEVENS
Institute of Technology

Geometric and Asymptotic Group Theory with Applications 2016

Shane O'Rourke

Cork Institute of Technology

A Combination Theorem for Affine Tree-Free Groups

Wednesday, June 15, 2016
Stevens Institute of Technology, Kidde 228
9:00 AM

Abstract:
Isometric actions on Λ-trees have been studied by several authors, including Morgan, Shalen, Chiswell, Bass, Kharlampovich, Miasnikov, Remeslennikov and Serbin. In particular Bass showed how isometric actions of (vertex) groups on Λ_0-trees can be combined to give an isometric action (on a $\mathbb{Z} \times \Lambda_0$-tree) of the fundamental group of an associated graph of groups, provided certain compatibility conditions are met. Notably, the hyperbolic lengths of the embedded images $\alpha_e(g)$, $\alpha_{\bar{e}}(g)$ of elements g of edge groups must match up.

Affine actions are actions by dilations: one requires $d(gx, gy) = a_g d(x, y)$ where a_g is an order-preserving group automorphism of Λ. In this talk we will show how certain combinations of groups can be equipped with an affine action on a Λ-tree. That is, if a graph of groups is given where the vertex groups have affine actions on Λ_0-trees, the fundamental group admits an affine action on a Λ-tree where $\Lambda = \mathbb{Z} \times \Lambda_0$, provided certain compatibility conditions are satisfied. Focusing on the case of free actions, we show that a large class of one-relator HNN extensions of free groups admit free affine actions on Λ-trees. Such HNN extensions cannot typically act freely by isometries because of the requirement that $\alpha_e(g)$ and $\alpha_{\bar{e}}(g)$ have the same hyperbolic length.

Using recent work by various authors, we also show that groups that admit a free affine action on a $\mathbb{Z}^n$-tree with no inverted line are locally relatively quasiconvex and relatively hyperbolic with nilpotent parabolic subgroups; they therefore have solvable word, conjugacy and isomorphism problems.