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On conjugacy separability of subdirect products

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Abstract:
Let $\mathcal{C}$ be a class of groups (e.g., all finite groups, all p-groups, etc.). A group G is said to be *residually-$\mathcal{C}$* if for any two distinct elements $x, y \in G$ there is a group $M \in \mathcal{C}$ and a homomorphism $\varphi : G \to M$ such that $\phi(x) \neq \phi(y)$ in M. Similarly, G is *$\mathcal{C}$-conjugacy separable* if for any two non-conjugate elements $x, y \in G$ there is $M \in \mathcal{C}$ and a homomorphism $\varphi : G \to M$ such that $\varphi(x)$ is not conjugate to $\varphi(y)$ in M.

In the talk we will discuss $\mathcal{C}$-conjugacy separability of subdirect products $G \leq F_1 \times F_2$, where F_i, $i = 1, 2$, are either free or hyperbolic. Let $N_1 := F_1 \cap G \lhd F_1$. The standard idea, originating from Mihajlova's trick, is that "bad" properties of the quotient F_1/N_1 transfer to "less bad" properties of the subdirect product G, and "good" properties of F_1/N_1 give rise to "even better" properties of G.

Following this philosophy, we will prove that if F_1/N_1 is not residually-$\mathcal{C}$ then G is not $\mathcal{C}$-conjugacy separable; on the other hand, if F_i, $i = 1, 2$, are free and all cyclic subgroups of F_1/N_1 are closed in the pro-$\mathcal{C}$-topology, then G is $\mathcal{C}$-conjugacy separable.

As a consequence, we deduce that for any prime p and the class $\mathcal{C}_p$ of all finite p-groups, a finite index subdirect product of two free groups is $\mathcal{C}_p$-conjugacy separable if and only if the index of G in $F_1 \times F_2$ is a power of p. In particular, if a subgroup of finite index in the direct product of two free groups is $\mathcal{C}_p$-conjugacy separable for at least two distinct primes p, then it must itself be a direct product of free groups. We will also discuss another criterion of $\mathcal{C}_p$-conjugacy separability with applications.