

# Geometric and Asymptotic Group Theory With Applications 2016

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# On CT and CSA Groups and Related Ideas

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#### Abstract:

A group is G commutative transitive or CT if commuting is transitive on non-trivial elements. A group G is CSA or conjugately separated abelian if maximal abelian subgroups are malnormal. These concepts have played a prominent role in the studies of fully residually free groups, limit groups and dicriminating groups. They were especially important in the solution to the Tarski problems. CSA always implies CT however the class of CSA groups is a proper subclass of the class of CT groups. For limit groups and finitely generated elementary free groups they are equivalent. In this paper we examine the relationship between the two concepts. In particular we show that a finite CSA group must be abelian. If G is CT then we prove that G is not CSA if and only if G contains a nonabelian subgroup  $G_0$  which contains a nontrivial abelian subgroup H that is normal in  $G_0$ . For K a field the group PSL(2, K) is never CSA but is CT if char(K) = 2 and for fields K of characteristic 0 where -1 is not a sum of two squares in K. For characteristic p, for an odd prime p, PSL(2, K) is never CT. Infinite CT groups G with a composition series and having no nontrivial normal abelian subgroup must be monolithic with monolith a simple nonabalian CT group. Further if a group G is monolithic with monolith N isomorphic to PSL(2, K) for a field K of characteristic 2 and G is CT then  $G \approx N$ .