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Amenability of Schreier graphs and strongly generic algorithms for the conjugacy problem

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Abstract:
In my talk I report about a recent joint work with Alexei Miasnikov and Armin Weiß. The starting point is that, frequently, the conjugacy problem in finitely generated amalgamated products and HNN extensions can be decided efficiently for elements which cannot be conjugated into the base groups. This observation asks for a bound on how many such elements there are. Such bounds can be derived using the theory of amenable graphs:

In this work we examine Schreier graphs of amalgamated products and HNN extensions. For an amalgamated product $G = H \star_A K$ with $[H : A] \geq [K : A] \geq 2$, the Schreier graph with respect to H or K turns out to be non-amenable if and only if $[H : A] \geq 3$. Moreover, for an HNN extension of the form $G = \langle H, b | bab^{-1} = \phi(a), a \in A\rangle$, we show that the Schreier graph of G with respect to the subgroup H is non-amenable if and only if $A \neq H \neq \phi(A)$.

As an application of these characterizations we show that under certain conditions the conjugacy problem in fundamental groups of finite graphs of groups with free abelian vertex groups can be solved in polynomial time on a strongly generic set. Furthermore, the conjugacy problem in groups with more than one end can be solved with a strongly generic algorithm which has essentially the same time complexity as the word problem. These are rather striking results as the word problem might be easy, but the conjugacy problem might be even undecidable. Finally, our results yield another proof that there is a strongly generic subset where the conjugacy problem of the Baumslag group $G_{1,2} = \langle a, t, b | bab^{-1} = t, tat^{-1} = a^2\rangle$ is decidable in polynomial time. The conjugacy problem $G_{1,2}$ is decidable, but it might be that the lower for the average case complexity is non-elementary.