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Exploding Pullbacks & Intersections within Free-Abelian times Free Groups

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Abstract:
The classic theory of Stallings describing subgroups of free groups as automata provides a very convenient (and fully algorithmic) way of understanding the intersection of two finitely generated subgroups $H_1, H_2 \leqslant F_n$. Namely, the automata associated to the intersection $H_1 \cap H_2$ is precisely the trimmed connected component containing the basepoint of the pullback of the automata associated to H_1 and H_2. This scenario immediately provides a finite basis for $H_1 \cap H_2$ (and thus Howson's property for free groups), as well as the classic Hanna Neumann bound for the rank of such an intersection:

$$\mathrm{rk}(H_1 \cap H_2) - 1 \leqslant 2(\mathrm{rk}\, H_1 - 1)(\mathrm{rk}\, H_2 - 1).$$

We extend the former description to subgroups of $\mathbb{Z}^m \times F_n$ by admitting abelian labels in the edges, and modifying consequently the folding process. This allows us to describe the automata associated to the intersection of two finitely generated subgroups $H_1, H_2 \leqslant \mathbb{Z}^m \times F_n$ as the one obtained from the enriched pullback of the free projections of H_1 and H_2, by "exploding" the edges according the abelian labels it has. Since these "explosions" can be infinite, we immediately obtain that (nondegenerated) free-abelian times free groups are not Howson. On the other side, when all the explosions are finite, it becomes clear that they can be finite *of unbounded size*. Thus, (even for finitely generated intersections) no "Hanna Neumann"-like inequality can be expected for any family containing nondegenerated free-abelian times free-groups. This is a joint work with Enric Ventura.